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Obsah

INTRODUCTION

You are going to read a new issue of magazine Usta ad Albim Bohemica, which freely follows the first issue of the X. number called The Beauty in Mathematics.

After three years, this issue moves this theme to the next level and shows a splendid nook of Mathematics. We can call it The Beauty in Mathematics II.

The authors, whose articles you are going to read in this issue, are not only important mathematicians but also postgraduate students in the field of the Mathematics, the Didactic of Mathematics and also the Didactic of Information and Communication Technologies. There are included articles from the Czech Republic, Slovakia, Poland and the United States of America.

This issue is made up of 23 contributions by the authors and teams of authors and you will find there for example:

Ján Gunčara in his article *Examples from Historical Mathematical Textbooks with Using GeoGebra* presents the beginnings of the important Italian secondary school and its project called La Nuova Geometria del Compasso Le construzioni di Lorenzo Mascherioni utilizzando il software GeoGebra. This article shows the possibilities of using GeoGebra in the historical – mathematical textbooks.

Pavel Tlustý and Lenka Činčurová contributed to the issue with the article called *Several Notes about the Harmonic Series.* It deals with the essential creation of convergence and divergence of series and also, it presents and analyses a number of practical examples that lead to the harmonic series.

Other interestinc article by Adam Płocki discusses *Lotto in Mathematical Tasks or Mathematics without Computing.* It shows some peculiar reasoning connected to mathematical problems, which were inspired by the hazardous game of LOTTO.

Rastislav Telgársky, the author of *Dominant Frequency Extraction* in his article mentions the extensive studies of the time series and speaks about the empirical approach towards the time series in general.

Alena Fleková and Bohumil Novák contributed the article *Inquiry – based Mathematics edication – a Challenge and a Chance.* They inform about a method that may lead to achievement of positive changes in Mathematic education and in natural sciences. Also, they mention the Fibonacci project and present several examples how to create an environment suitable for use of inquiry – based methods in primary school education context.

One of the other interesting articles is *Numeral Blocks as an Educational Tool for Development of Mathematical Thinking in Preschool Preparation* written by Jan Melichar. In this article, the author presents the framework of ESF We Manage to Do It Together – CZ.1.07.1.-2.00/08.0105, where there is proposed the aid for the mathematical thinking development. Also, the author describes the methodology of this aid.

Linear Programming and Heuristic Strategies is the headline of the next article by Jan Kopka, George Feissner and Jiřina Ondrušová. This collective of authors analyses the use of heuristic strategies and describe the usage of this strategy by the mathematicians in practice. Also, they depict the importance of introducing this strategy to the schools.

Štefan Tkačik wrote an article called Galileo's Paradox, which is dedicated to the surprising properties of infinite sets in Galileo's revolutionary work work "Discourses e Dimostrazioni matematiche intorno à due nuoue Scienze", published in 1638 in Leida. One of the aims of this article was to show the clash that appears the very first chapter of this work and also, to present the conclusion made later on by Cantor.

I believe this issue will be interesting for you and that it will serve as a source of new inspiration for your work.

I wish you a pleasant discovering in The Beauty of Mathematics II.

Lukáš Círus

INFORMATION AND COMMUNICATION TECHNOLOGIES AND THE DEVELOPMENT OF MATHEMATICAL SKILLS OF CHILDREN WITH DISABILITIES

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Abstract:
Computer games and computer learning software products help children with disabilities develop their skills in different subjects. Our contribution is focused on the development of mathematical skills of handicapped children, depressed children, and children with mild mental disability and sense disability. Using information and communication technologies, it's possible for these children to develop their motile, cognitive processes, concentration, and eliminate psychical issues, which coming from their disability.

Keywords: Information and Communication Technologies, children with disabilities, development of mathematical skills
MESC: U70, U10

1 Introduction

Computer game considers the kind of activities that not only provides entertainment, distraction, pleasure, satisfaction, but is also an important educational tool. In the computer game, children develop cognitive processes such as perception, memory, thought operations, creative thinking and concentration of attention. The move, drill, practicing rough and gentle motile are leading to the intensification of learning of children.

The game has a significant impact on the socialization of the child to develop his skills, abilities to shape his personality. It provides release him from any internal unrest and experiencing joy. Game teaches the child to win play and compete.

Computer games as well as other games give the child positive survivals and involved a great deal to the development of his personality[1]. They are important educational and training resources. Computer games can play an important role in tackling mental health problems of the child, which is particularly important for children with disabilities. We focused, therefore, on children with disabilities, children with poor health, on children with mild intellectual disabilities and sensory impairments.

[1] ORTANČÍKOVÁ, H. Rozdelenie počítačových hier. In: Informatika v škole a praxi. 4. ročník konferencie s medzinárodnou účasťou, 16.-18.9.2008, Ružomberok, s. 230 – 234, ISBN 978 – 80 – 8084 – 362 – 5

Computer games with educational computer programs are involved in the development of these children in various subjects such as the Slovak language, natural science, mathematics, and others. We focused another text, in the acquisition and development of mathematical skills of children with disabilities.

2 Information and communication technologies and the development of mathematical skills of children with disabilities

We can to develop mathematical skills for preschool age children with disabilities to apply computer game Shape Drop. To play games Shape Drop must use the cursor arrows and insert individual geometric shapes into the holes. With this game is being developed for preschooler distinction of shapes, sizes, orientation in space and attention. [2]

Fig. 1 Shape Drop
(http://logicke.1001hry.cz/shape-drop.html)

For preschool age children with health impairments may apply computer game Welcome to robots. Robotics Malik and his friends seek in the computer game to establish the ferrous scrap useful things for his home. One of the friends Perák is trying to put together pieces of scrap. The role of the child is to help him with the mouse. Object to form is on the top of the screen and below him are placed pieces of scrap. Child makes the object with these pieces. Child uses his imagination, fantasy and knowledge of geometric shapes[3]. Game reinforces the child self-confidence allows him to develop self-realization.

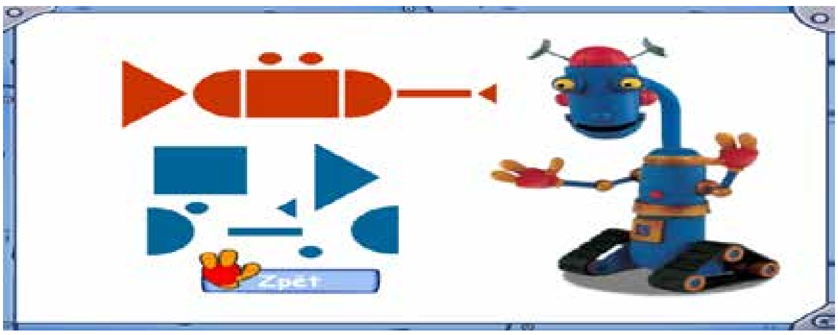


Fig. 2 Welcome to robots
(http://www.minimaxcz.tv/minigame.php?name=littlerobotscz)

[2] http://logicke.1001hry.cz/shape-drop.html
[3] http://www.minimaxcz.tv/minigame.php?name=littlerobotscz

To develop mathematical skills of a child younger and older school age with disabilities can use a computer game Alík - Cheerful mathematics. Computer game Alík - Cheerful mathematics is on CD and deal only with mathematical tasks. Game is for children first to third grade. This focus is adapted throughout the application environment. The controls are very simple. The amount of interesting images, animation, graphic effects resulting child very natural way to math homework. Child can solve tasks in the numerical domain into ten to twenty or a hundred. For the right solution to each of the games the child acquires money, for which you can buy toys in Alikov toy stores.

After launching the application, the splash screen with the dog Alik, which explains the rules of the game. The child chooses player by clicking on any of the displayed piglets. Child writes their name to tabs and select a numeric field, to know how to count. Deleting a player runs with the hat. The screen to select the player to click on the baby rocking horse can get into the interpretation of the toy. It's actually a screen to choose their own games. If he wants to buy some toys, child used to handle on the door of the shop. In it are toys with price tags. As in real life, and here a child can choose only those toys which has hoarded money. Click on the scooter dog dare Alík bought toys imaginary child's room. In it can be with toys "play". Click one of these toy demonstrate some animation accompanied by sound.

When working with the games need to seriously listen assignments. If the child did not understand assignment, you can repeat it by clicking on the lifeline. "Wooden" arrow with traces used to go to the previous screen.

For multimedia CD Alík - Cheerful mathematics are the following tasks:

- Snake game contains examples paved path that leads to player Alik’s house. Each stone is an example that child calculated by clicking on the correct number at the bottom of the application. Confirms the result is checked. If you want to fix it, click on the icon tangle.
- The game Dragon has child count the dragon head to wake up after the bell rings. After three successful attempts it followed by reward. It's easy game suitable for young children.
- Memory is the classic game of finding pairs. In this case, the pair are examples and their results. This game requires concentration and may sweat even older school age children.
- Formula is a very clever design game in which the child Sovereign correct numbers in the examples pushes his athletes to sweet reward. An example must be calculated as quickly as possible.
- Bear with bubble blower is another game that is focused on speed. Child is designed to develop cracks bubbles from smallest number to largest or outcome. The game is suitable for children older school age.
- In the game shooting range for the child's role calculates results Alik hits the target. Each correct result is a reward.
- The aircraft is a game in which the child counts examples in the workbook. Child can choose between adding or subtracting and also whether to add a number or an equal sign, inequality.
- The game Weight presents tasks aimed at comparing the numbers or results on both sides of the instrument. Reward following five successful attempts.
- The game Dog strongman child sovereign marks examples to Alik help in the fight pudding. If victorious Alík, child receives remuneration. [4]

CD fun for the child develops math skills, concentration, attention and speed of response.

Child Kon-Zen is additional games to develop mathematical skills of school-age. Kon-Zen program is designed for practicing concentration of attention and thinking through visual games.

[4] http://www.czs-svmi.sk/dokumenty/multimedia.pdf

Three types of games (Couples, Mapping, and Lightning) allow independent adjustment of difficulty for each of them. The easiest level contains geometric shapes, intermediate level semicircles and quarter circles, the last eighth and sixteen circles. The program allows practicing letters and numbers, the child develops visual perception, spatial imagination, thinking, attention, speed and short-term memory. The game is controlled with the mouse, touch screens, one or more buttons to move the cursor using the automatic (scanning). The program can be used by children from five years old, people with impaired visual perception, problems with reading and spelling, but also older people for practicing mind power and those who want to improve the ability to concentrate attention. [5]

Online baby Sudoku the size of 4x4 is a game that develops in children older school age logical thinking.[6] It is possible to select the number and click the left mouse button, drag it to the empty box in Sudoku. The other way is to click on the blank rectangle in a circle show numbers that can be inserted into the Sudoku. Wrong number is removed so that it clicks the left mouse button and, while holding the pull out table Sudoku, or you can move it to another location in the table. In the game you can turn on / off control of bad moves.

The objective of computer games Counting apples is to teach a child to calculate an example, that monkey could pass a basket of apples on the other side of the river. With the correct calculation goes smoothly, but if bad example calculates fall into the trap of crocodile (http://www.play-kidsgames.com/games/apples/savetheApples.htm). The game is in English. Operated mouse. The child develop math skills, speed of response, the concentration of attention.

Fig. 3 Counting apples
(http://www.playkidsgames.com/games/apples/savetheApples.htm)

Numbers can also be applied to Bork builder. Child controls the game using the mouse, count the correct number of objects in the picture and the number of moves in the boxes. When he makes a mistake game not let him play next level. Game is in English.[7]

The game develops accurate perception, concentration of attention.

Game Apple Tree develops a child's perception, speed and accuracy of responses, attention, soft motile skills. Use the cursor arrow saves a baby fox with apples falling from the tree to the basket. In the upper right corner appears the number of fallen apples in fox basket.[8]

For a child under school age and older who are disabled disease can develop mathematical skills to use next computer games.

[5] http://www.petit-os.cz / progr_lifetool.php # Kon_Zen
[6] http://www.omalovanky.sk/online-detske-sudoku/167-hry-pre-deti/online-sudoku/1835-online-detske-sudoku
[7] http://www.sproutonline.com/games/spuds-counting-game
[8] http://www.extrahry.sk/hra/apple-tree-game

The game distances - What carries Brumík? child chooses first picture, which he likes. The picture is incomplete and therefore for the child is to be supplemented so that it is useful. Has a number from one to eleven, which must correctly connect the dots in numerical range of the mouse. The result is that when the correct procedure appears complete picture.[9] The game is controlled with the mouse. Bringing a sick child to the joy of the final image, removes tension and restlessness, which is in itself. It is suitable for children younger school age separately for freshmen and sophomore.

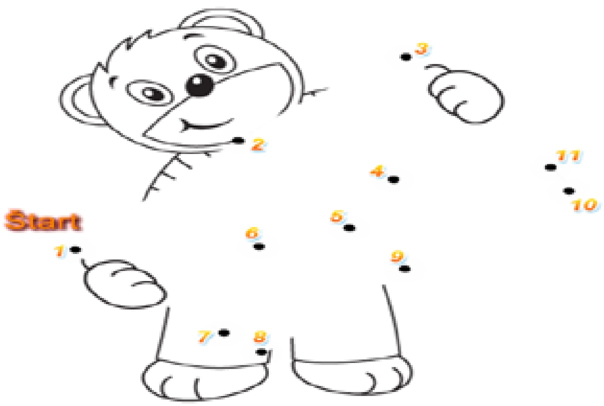


Fig. 4 Distances - What carries Brumík?
(http://www.brumik.sk/clanek/zobrazit-clanek/84-co-brumik-nese/)

Computer game MICKEY MOUSE is designed for children younger school age. Mickey Mouse writes on the blackboard example of a child trying to calculate it. Child presents the result of his calculation clicking on the number that is the bottom of the screen. Use the check shall verify the accuracy of the result. If the result is not correct Mickey allow repair and give another example. Examples are 25 and in the lower left corner of the figure appear which envisages an example of a child. [10]The game develops the child's thinking, brings him joy of a successful outcome.

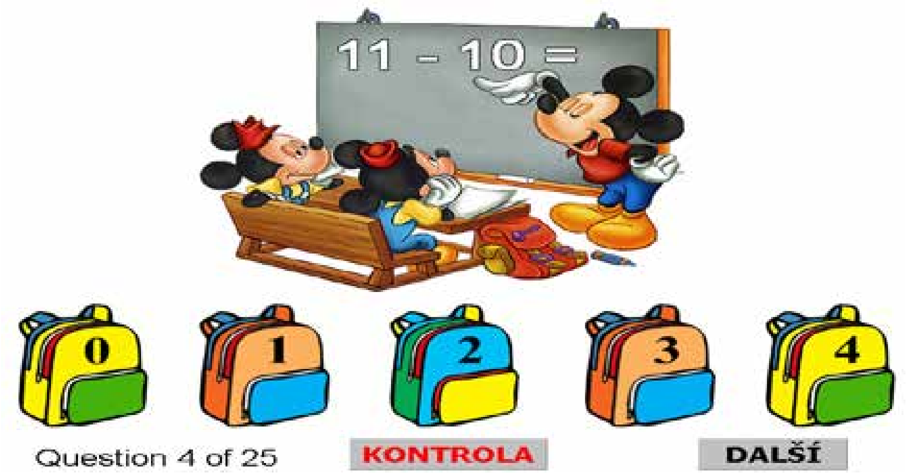


Fig. 5 Mickey Mouse
(http://www.pripravy.estranky.cz/clanky/prvni-trida/pocitame-s-mickey-mousem.html)

[9] http://www.brumik.sk/clanek/zobrazit-clanek/84-co-brumik-nese/
[10] http://www.pripravy.estranky.cz/clanky/prvni-trida/pocitame-s-mickey-mousem.html

Computer game first blink, then click has three difficulty levels. The first is for children of younger school age who attend higher grades of primary school first grade, second and third level is for older school-age children. The role of the child is quickly read the instructions and uses the mouse to respond to what is written. Bottom left of the screen will appear as percentage points. The maximum number of points on the first level of difficulty is 18.[11] Computer game with a child develops memory, reaction speed, concentration of attention, examines his knowledge of geometry that is geometric shapes and elements, recognize the colours. Game contributes to the development of autonomy and self-confidence.

Remind even the computer games for a child under school age and older with health impairments, which also develop their math skills.

In the computer game Tangram 2 is for the child to pass geometric shapes on the right side of the screen to find matching shapes in the picture. Use points located at the corners of a child to manipulate shapes. The game is controlled by mouse.[12] Computer game with a child develops orientation in space, imagination, attention. He brings a sense of full fillment, enhances his self-esteem and confidence.

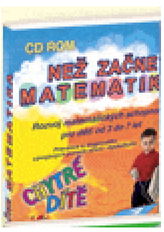
The role of the child in the computer game Count the Cubes! is to count the cubes and click your mouse on the appropriate number in the right side of the screen. Click Submit to confirm accuracy of the result is that thousands of items have been added to the score. If child make mistake, the thousand points are deducted. The game consists of ten tasks, in which the intensity increases. Press the Clear key can erase bad result and count again. [13]

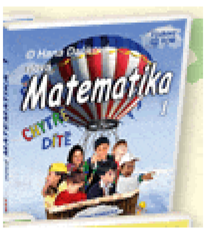
Also requires some patience and peace. A good result brings the child satisfaction and increases self-confidence.[14]

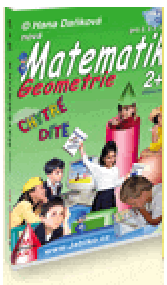
3 Information and communication technologies and the development of mathematical skills of children with mild intellectual disabilities and sensory disabilities

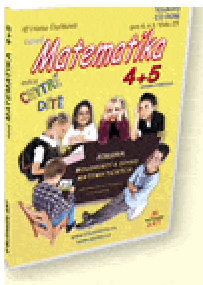
For a child with mild mental disabilities is an appropriate application of computer programs relating to the acquisition and development of mathematical skills appropriate to their disability.

Through play smart kid - Mathematics get an idea of the natural numbers, it is also aimed at comparing numbers, sorting geometric shapes, etc. They develop skills that are the basis of mathematical cognition and logical relationships.









[11] http://www.minimax.hu/flash/nyomi/nyomi.php?lang=CZ
[12] http://www.kibagames.com/Game/Tangram_2
[13] http://www.123bee.com/play/count_the_cubes/17971.html
[14] HABŠUDOVÁ, M., VAŇKOVÁ, J., BEDNÁŘÍKOVÁ, A. 2012. *Počítačové hry a ich úloha v somatopédii.* Ružomberok Verbum 2012, s.105, ISBN 978-80-8084-964-1

Mathematics 2 +3 CD is designed for second and third grade of primary school, the focus is on geometry. Teacher Bajtík explains geometry to a greater extent, such as school curricula. For example, the information on line is very important to emphasize that we cannot measure, familiarity with angles associated with the triangle, etc. Subject matter of the second and third class are related. As a child learns to understand and measure a segment needs to know to what is to do. Knowledge about the length of a line segment is fixed, for example by measuring the length of your room. Blue Bajtík present their roles to practice. There are also included difficult task quiz and Olympics. The controls are simple, the child chooses one Bajtík. On the lower buttons of the same colour as Bajtík chooses role. Right circular buttons differentiate role. In each task is help - Bajtík with stool. After solving task child must to ring to Bajtík. Arrow selects the next example. On blackboard was writing the correct number of solved examples.

Here are also role models with yellow Bajtíkom that children choose according to their interests. It appears Model containing addition, multiplication, hours of operation and construction. Teaching jobs consist of being a teacher in Bajtík animated clip explaining new concepts and relationships. CD helps children understand mathematics and ease of learning difficulties in mathematics.[15]

The pupil is a specialized program for children with mental retardation and mild degree is also suitable for children attending primary education. It is designed for practicing mathematics and the development of children's mathematical knowledge. The aim of the program is to bring the numbers of mentally handicapped children, their size and learn their individual numbers added together. Software is not complicated, is designed to attract the attention of students and motivate them to work.

It is divided into two parts:

1. 1 Guess number (recognition of numbers in the range 0-20, acquisition value numbers, comparing numbers through the display of the number of balls, choosing the right number of balls).
2. 2 Guess sum (sum of two numbers, comparing the sum of two numbers, counting the two numbers and choosing the right result, familiarity with calculator).

Both parts of the program are processed audio, which will provide a better understanding of the assignment. Attention mentally disabled is not stable; because the software is compiled to work with him was easy for them and suitable for the disabled. The mentally handicapped is narrowed attention span, the number of elements that can be perceived. Theoretical knowledge and practical experience say that the time limit for mental alertness is impaired 15 to 20 minutes, so when creating software was necessary to adapt it so that pupils work with him lasted more than 20-25 minutes. This time is dependent on the ability and motivation of children. Tiger Lea, who accompanies the student the whole program and tells him what to do, is the motivating factor.

The software was developed in Borland Delphi 7, which is a visual programming environment that is used to develop applications for the Windows OS. Individual images were edited in Paint environments, ACDSee 5.0 and HP Image Editor. The sounds were recorded in the program Audacity.

[15] http://www.jablko.cz/chytredite

The program is run simply by clicking on the icon. It consists of a number of forms, which are interlinked. The forms are visually treated to adopt a child at first sight with sound recordings. Sound recording is triggered by clicking on the trumpet.

After starting the program in the form displays written text, edit, character Lea and two buttons, button the end key and the sound recording. At the beginning of the tiger Leo child present and says that they will play together. She asks him to write his name and clicked on the image. If a child does not write his name, he cannot do anything. Click on a picture while new button, purple flashing arrow that can go on in the program Guess the number Guess sum. After running the form is loaded image of Leo's cottage, two of which lead the way. Leo stands at a crossroads, who decides, after which the path is chosen. Each of the paths will lead a child to a tree, the crowns of which are buttons that link to other parts of the software. In the left tree crown are colour numbers, meaning Guess part number. The crown of the tree is real numbers with a plus sign and equal, which is part of the Guess sum. There is a text that speaks to the child chose the game you will play with tiger Leo. Click on the crown of the tree chooses. Sound recording child explain in more detail what you can do.

The calculator is very similar to the classic calculator. The difference is in the fact that it contains only counting. The calculator contains buttons with the numbers 0-9, the plus, equals, delete and end button, which closes the calculator. Figure whispering Lea is equal to the calculator on the form.[16]

For a child who is visually impaired can be used to develop mathematical skills of the Island unforgettable second year of elementary Mathematics. Program unforgettable island is primarily used in mainstream schools for pupils intact. Using a special magnifying programs, possibly using different specific assistive devices can also be used the program for students with visual impairments, for example, visually impaired pupils. The island is unforgettable educational program aimed at mastering the second year of elementary school mathematics. CD contains everything second year elementary school pupil should know mathematics. Through the program, children playfully acquainted with the mathematics curriculum and thus unattractive mathematics curriculum for them is more attractive. Mr island is unforgettable Captain Kormorán, which shows the way around the island. When solving problems, the children do not have to worry about mistakes, because every wrong answer immediately corrects his parrot Repeat and explain where the child made a mistake.

The program also includes additional functions such as a notebook with the ability to print, the Internet connection, files, games for relaxation, background music and other additional functions that make the program even more attractive and also more attractive for children with visual impairments. Upon completion of each task - "Test" program shows the evaluation of success through animated graph. The evaluation shows the ratio of number of correct answers, percentage and total duration of all tasks. Demo version of Island unforgettable second year elementary school children MATHEMATICS we found at http://www.lumi.sk/lumi_sk/pozri_ostrov_mat2.htm.

A child with hearing impairments can be applied to the development of mathematical skills such computer programs:

[16] SZARKOVÁ, E. 2007. *Edukačný softvér pre mentálne retardovaných žiakov.*2007, Univerzita PJŠ Prírodovedecká fakulta Košice, bakalárska práca, s.61

In the first year of a child can work with tutorials for Mathematics I. year - we learn with Ferdo. The program contains a number, numeral, numeric range to 10, comparing to 10, addition and subtraction within 10 to 20 numeric range, comparing to 20, addition and subtraction to 20 without going through a transition as ten, geometry - exploring and determining the shapes.[17]

Tutorial Mathematics II. year - we learn with Ferdo contains numeric range of up to 100, comparing numbers to 100, 100 to counting up to ten, counting by tens to 100, counting to 100 with up to ten, geometry, transfers measures of length, clock multiplication and division numbers 1 to 5 The program evaluates the correct and incorrect solutions, after counting follows reward for children, fairy tale about Ferdo.[18]

4 Conclusion

Computer games and computer learning programs help children with disabilities develop their individual skills in our case, mathematical skills, develop their motile skills, imagination, perception, expand their vocabulary. Thanks to them, children can travel around the world, discover the beauty of nature, and communicate with similarly disabled children living in other countries, helping them to find a sense of usefulness and self-ful fillment. Children at work with computer gain self-confidence and self-assurance, often calm down, eliminating their internal tensions and more seek to communicate, to obtain further information. Computer programs give them the chance to learn and often the primary and any only possible literacy in contrast to unaffected individuals who acquire computer literacy through secondary.

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Internet resources:

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[17] http :/ / www.infovek.sk/predmety/matem/index.php?k=33
[18] BEDNÁŘÍKOVÁ, A., VAŇKOVÁ, J. 2011. *Informačné a komunikačné technológie pomáhajú deťom mladšieho školského veku s postihnutím.* Ružomberok Verbum 2011, s.108, ISBN 978-80-8084-815-6

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DROPPING A PART OF THE CONDITION – AN EFFECTIVE HEURISTIC STRATEGY

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Faculty of Science UJEP, Ústí nad Labem1,

Abstract:
This article describes a heuristic strategy of removing conditions and illustrates it by four school math examples. These examples are efficiently solved while using such strategy.
Keywords: dropping a part of the condition, problem-solving strategy, problem solving
MESC: D50

1 Introduction

Developing strategies facilitating mathematical problem-solving dates back to the ancient times and brought the discipline of heuristics to life.

Heuristic methods, heuristic strategies, or simply heuristics are rules of the thumb for making progress within complicated problems (Polya, 2004). They are general suggestions of a strategy designed to help while we solve tasks (Schoenfeld, 1985). For Bruner (1999), they are methods and strategies helpful in problem-solving. Throughout the history, recognised mathematicians have used and developed solving strategies. Let us name but a few: Pappus, Descartes, Leibniz and Bolzano. Strategies discussed in this article are actual instruments helping find the path from formulation of the problem to its solution.

A full range of useful strategies commonly used by mathematicians while problem-solving is known; it would be of the highest usefulness if some of these entered school maths. A school teacher should develop his/her students' know-how, their ability to reason together with encouraging their creative thinking (Polya, 2004). We shall list a few essential strategies with their names leading us to the core: experimentation, generalization and concretization, analogy, dropping a condition, auxiliary element, dividing a problem into several sub-problems, working backwards.

Kopka (2013) has offered a more detailed description in his book. A similar classification of heuristic strategies is presented by Fan and Zhu (2007), to name but some: "Guess and check", "Solve a part of the problem", "Draw a diagram", "Make a systematic list".

Good teachers use some of the above mentioned strategies intuitively. The path to such ways would be via dissatisfaction with standard problem-solving through the means of standard methods when they were pupils and students themselves and later became teachers. We will present one of these heuristics strategies, namely dropping a part of the condition.

2 Dropping a part of the condition

The task given includes several conditions set. If we are inapt to meet all expressed conditions, we may, together with Zeitz (2007), query: „What makes this problem so difficult?“, and should we succeed in pinpointing the crucially challenging condition, we might try and drop the one. If this helps us solve thus weakened problem, we return to the condition dropped and try solving the task.

Polya has also described partially the strategy of Dropping a part of the condition in his book on problem-solving (Polya, 2004).

Other aspects of this heuristic strategy are described in Kopka's (2013) book. Terence Tao (2010) introduces the ways to change a problem significantly and therefore to facilitate solving the problem. Tao shows "more aggressive" sorts of strategies via which we adapt the problem, e.g. negating the objective or the very condition dropping.

Let us proceed with illustrating the above mentioned strategy with several tasks.

It is also needed to be said that many a task presented here may well be solved in a number of different ways, including the standard way or other heuristic strategy.

Naturally, we present solutions achieved by using dropping a part of the condition. Its expediency is to be best noticed where there it is the most efficient path to reaching the result, or, may the case be, the only way to solve the problem; the following task (no. 1) is of such an example.

Task 1

Assignment: Inscribe a square $KLMN$ in a given triangle ABC. Two vertices (K, L) of the square should be on the base AB of the triangle, the two other vertices (M, N) of the square on the two other sides of the triangle, one on each. (Polya, 2004, p. 23)

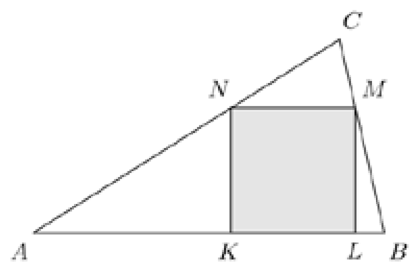

Fig. 1

Solution: It is unlikely we succeed in constructing a square executing all conditions. If we drop the "the M lies on the BC side" condition, we may find constructing such a $K_1L_1M_1N_1$ square with ease (see fig. 2). We inscribe it in the BAC angle. The square we search is to be homothetic to that, with homothetic centre in the A point. The point M is the point of intersection of the ray $AM_1$with the BC side.

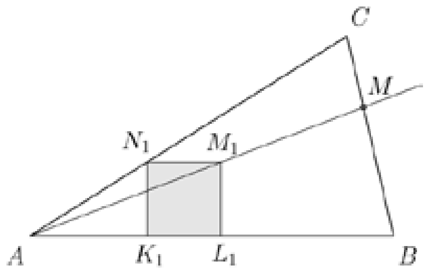

Fig. 2

Note: Should the pupils not be familiar with homothetic transformation, we could reach the solution by experimentation. More squares may be entered into the BAC angle (see fig. 3). All points M_i constructed are apparently placed on the ray. We shall make the statement general.

Conjecture: All M_i vertices of $K_iL_iM_iN_i$ squares are placed on the ray with initial point A. Intersection of the ray AM_1 with the BC side is the vertex M of the sought square.

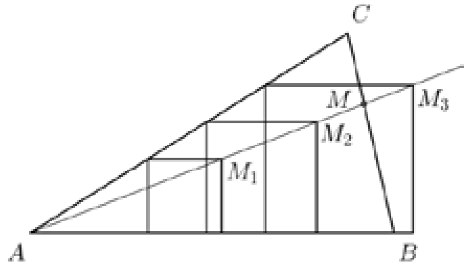

Fig. 3

The dropping a part of the condition strategy can well be used within some word problem, as seen in the following problem.

Task 2

Assignment: Some of the theatre tickets cost CZK 11 and some were sold for CZK 8. How many of each sort were there, if the totalling price went up to CZK 965 with 97 items?

Solution: Let us omit the amount of tickets sold in our following consideration. Thus we obtain a new task of which the solving we shall transform into solving an corresponding with diaphonic equation, with x standing for tickets at the price of CZK 11, y being the number of CZK-8-tickets, thus we apply:

$$11x + 8y = 965 \qquad (1)$$

The equation may be solved in two ways.

The first procedure is ideal for gifted students and leads to defining a pair of generators to integral solutions x and y.

$$y = \frac{965 - 11x}{8}$$

$$y = \frac{968 - 3 - 8x - 3x}{8}$$

$$y = 121 - x - 3 \cdot \frac{1 + x}{8}$$

Now we shall introduce the parameter $k \in \mathbb{Z}$.

$$\frac{1 + x}{8} = k \Rightarrow x = 8k - 1$$

When substituted, we reach

$$y = 121 - (8k - 1) - 3 \cdot \frac{1 + 8k - 1}{8}$$

$$y = 122 - 11k$$

The solution of the diaphonic equation (1) is a couple of numbers reading as follows

$$x = 8k - 1$$

$$y = 122 - 11k.$$

We shall organize particular results for individual k into a table.

k	1	2	3	4	5	6	7	**8**	9	10	11
x	7	15	23	31	39	47	55	**63**	72	79	87
y	111	100	89	78	67	56	45	**34**	23	12	1
total	118	115	112	109	106	103	100	**97**	94	91	88

Tab. 1

All solutions in the set of positive integers have been placed in table 1. Clearly, if we take the now dropped condition (totalling number of tickets is 97) into consideration, the task has successfully been resolved.

While following the second procedure of solving the equation (1), we shall be using a spreadsheet application, displaying the value of y in the second column as the following formula

$$y = \frac{965 - 11x}{8}$$

x	y	$x + y$
1	119,25	120,25
2	117,875	119,875
3	116,5	119,5
⋮	⋮	⋮
61	36,75	97,75
62	35,375	97,375
63	**34**	**97**

Tab. 2

Looking at table 2 we realize that on the last line we went back to the previously dropped condition (totalling number of tickets is 97) and reached a satisfactory solution.

Answer: 63 theatre tickets at CZK 11 were sold and 34 tickets at the price of CZK 8.

The dropping a part of the condition strategy is often used in school mathematics, without being named or realized. The following task aptly serves the purpose of illustrating the point argued.

Task 3

Assignment: We are given three points A, B, and C. Draw a line through A which passes between B and C and is at equal distances from B and C. (The problem has been taken from Polya, 2004, p. 73)

Solution: Assume A, B, C aren't collinear. If we try to solve this task in the trial-error way, we could consider an axis of the angle BAC to be a solution. Uprooting such a thought is easily done if we select a congruous set of points. Let us drop the condition of the straight line intersecting A and let us look for all lines with the same distance from B and C.

a) One of these lines is to be e.g. the axis of the BC line segment. The crucial part here is that the axis intersects the S centre of the line segment. It now is worth realizing that every line intersecting the centre of the BC segment is in the same distance from B and C. Such finding results from the identity of triangles, as we shall see below. Let us now apply the previously dropped condition and the line intersecting the centre of the BC segment will now intersect A. And now we go on to study fig. 4.

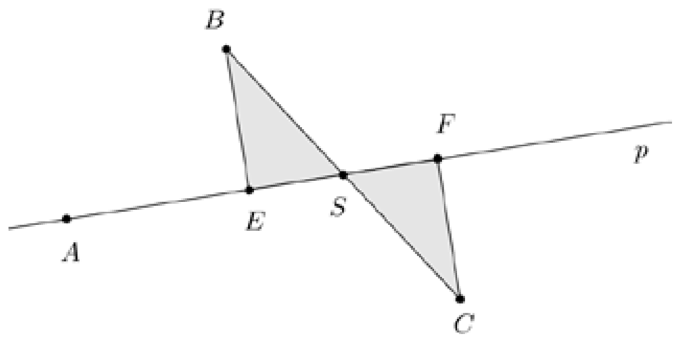

Fig. 4

To make the p line the one we have been seeking, there is a need for the validity of the following: $|BE| = |CF|$. Also the BE and CF segments are undisputedly perpendicular to p. For $|BS| = |CS|$ and $\sphericalangle BSE = \sphericalangle CSF$ and $\sphericalangle BES = \sphericalangle CFS$, the SBE and SCF triangles are identical and therefore segments line segments BE and CF are of the same size.

b) A different straight line of the same distance from the B and C points is an arbitrary q parallel with a BC line.

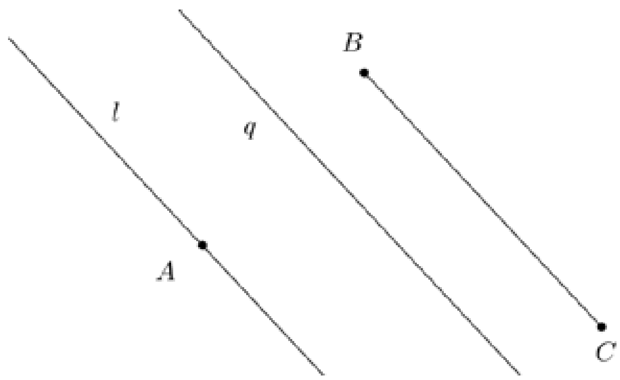

Fig. 5

As it becomes obvious, if the q line is parallel to the BC line, we read:

$$\mathrm{d}(B, q) = \mathrm{d}(C, q).$$

We shall now restore the dropped condition back to the task-solving and construct an l line, intersecting A and paralleling the BC line.

Answer: There are two ways to solve the problem. One is a line intersecting A and the centre of the BC line segment, the other solution is a line intersecting A and is parallel with the BC line.

The following and last task should be named Condition substitution. While solving the task, we drop one condition to substitute it with another one. The problem dates back to the 15th century (see Kopka, 1999). The here described way of its solution clearly shows the elegance with which mathematicians were able to solve tasks without having today´s school maths possibilities – equations.

Task 4

Assignment: The fish head weighs $\frac{1}{3}$ of the whole fish, its tail weighs $\frac{1}{4}$ of the whole fish, and its full body weighs 30 oz (see fig. 6). How much does the whole fish weigh?

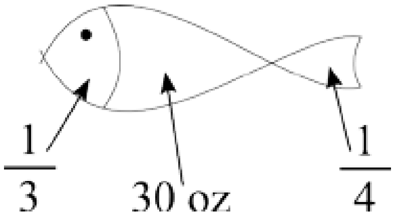

Fig. 6

Solution: Let us drop the condition of the fish weighing 30 ounces; without it, the problem cannot be solved. We shall substitute it with a different condition – and speak of a different fish: the whole one weighing 12 ounces. It may be obvious why 12 – it is the least common multiple of 3 and 4. The new fish has a head of 4 ounces and a tail weighing 3 ounces. The body weight is 5 ounces. The fish in our task is rather "similar" to this fish; its proportions are the same. Its body is six times heavier, the head and tail must also be six times heavier.

Answer: The fish weighs 72 oz.

3. Conclusion

To sum up let us express the conviction of ours that dropping a part of the condition strategy is useful and may be an effective way of solving school math problems for some pupils.

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THE INTRODUCTION OF MODERN TECHNOLOGIES INTO THE PRIMARY SCHOOL CURRICULUM

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Abstract:
The paper focuses on new trends in the introduction of Modern Technologies to the first grade of Primary school. It summarizes the efforts since the year 2000 to the present state *School 21*. There is a specific example of using the modern technologies in the classroom during the Math lesson.
Key words: ICT, Education, Math
MESC: U70, U10

1 Introduction

Information and communication technologies and the Internet are a phenomenon of our time. We come across them every day and they instantly affect our lives. In the history of mankind we can only with difficulties find other inventions and technologies that could have applied as quickly and significantly in everyday life as ICT. The society is gradually turning into an information society which is absolutely ICT dependent. There is a new, young "digital" generation growing that takes ICT as a natural part of their world. Our children can not imagine a world without ICT so there is an urgent need for everyone to learn how to operate ICT so they are used meaningfully and effectively to avoid any possible threat that arises from its usage and to minimize any intentional misuse. A very important role holds a Primary School that equips each student with necessary competencies for their further life.

2 The 21st century School

The plan continues with the intentions of the State Information Policy from 1999, on the basis of the state support in 2000 – 2006, by realizing a concept known as SIPVZ (the SIPVZ Concept). Currently, there exist a major difference between the technology application in a private life of each child and the technology practical use in education. Also, the lack of competences and teachers' qualification in technology use lead to the problems in the realization of other strategic goals. These aims and the technical development are taken into considerations in educational field.

- A major priority of the 21st century School is the optimal use of technology in education.
- The basic cornerstones of the 21st century School are:
- A creation, sharing and access of digital content

- An extension of a learning environment by the use of online services and social networks
- A feedback that will help to improve the schools
- A vision for Czech Education in the 21st century.

The main focus of the 21st century education is to offer pupils more fun and fewer stereotypes form of teaching and thus increase their motivation to learn and involve them more into the learning process. The pupils are no longer just passive listeners but they help to create and actively participate in the learning process. We change the 20th century knowledge into the 21st century skills. Education finds itself in a new, digital century and the teachers begin to acquire the student-centered approaches and to focus on the student’s skills and competencies. The pupils of the 21st century are called a “digital generation” because they are surrounded by the world of multimedia since they were born. It is difficult for teachers to hold their attention by a pencil and a chalk. The modern technologies and the digital content should help the teachers to enrich textbooks and worksheets with multimedia and with new forms of pupils’ involvement. The societies slowly but surely convert from print to the digital age.

The teacher of the 21st century is no longer the focal person in the class. Teaching is focused towards the pupils, their individual needs, skills and competences more than to an educational institution. The process of teaching is concentrated on the learners so they can feel as a part of education and the teacher gets the role of a facilitator and moderator. This approach forces the pupils to feel responsible for what they learn and to develop skills for lifelong learning and development with regard to the constantly changing environment.

The teacher’s assessment changes to a pupil’s self-esteem and self-evaluation. It is necessary that the students practiced giving and receiving feedback on their own or the others performance. Modern products and software tools support a swift feedback as well as an individual or mutual evaluation of work. They help to create a discussion and a regular feedback milieu. There are changes even in the curriculum. The Framework Educational Programs have developed the School Educational Programs that are differentiated and personalized; they reflect the needs of schools, teachers, pupils, parents and public. The aim is to discover ability in each student and develop it with the help of modern technologies that will enable him to a deeper and more detailed mastery of skills and competencies in areas that are of his interest.

The instructive teaching approach when the teacher leads the activities in the class and the learning process is mainly inferior to drills is being substituted or replaced by teamwork or a project learning; knowledge and skills are acquired indirectly through variety of educational activities and errors are not such a problem. The role of the teacher as a “facilitator” is irreplaceable and both approaches can be met in practice now.

The passive learning → the active learning

The teacher plans such activities that promote an active approach to the subject. Searching, sorting and comparing the information and their practical application. The art of working with texts, maps and other materials are part of every topic.

The teacher centered teaching → the student centered teaching

A pupil is in the center of all the activities carried out in the class. The teacher leads pupils to become independent; to participate actively; to be creative and to cooperate. The teacher is not the only source of information.

Individual work → group work

When performing a task the pupils are not working individually. The interactive learning process leads him to team work in education as well as sharing ideas and opinions.

Undifferentiated approach to pupils → differentiated approach to pupils

The teacher that uses modern technologies has the opportunities to split teaching methods according to the needs and skills of the student. The teacher helps to let the students know about their abilities and apply them in topics of their interest.

The teacher assessment → self- evaluation

The current technologies give students immediate feedback. The pupil realizes his mistake immediately and has a chance to evaluate his work.

The subjects secluded → the cross-curricular relationships

The teacher and the pupils have got the great opportunity to convey and understand the global context across the curriculum more quickly and vividly.

A single source of information → a number of sources of information

The new technologies allow each of us to access information globally and have a cohesive overview of the selected topic. The teacher does not use the textbooks only. It helps the student with the classification of the gained information and its sources.

The textbooks and worksheets (the printed media) → the multimedia

The interactive teaching approach is based on the use of multimedia – audio, video, flash animation, object manipulation and other digital tools. The experience of this kind of method helps the student to remember the topic and to understand better.

The school orientation → the society orientation

A supportive school environment is that in which learners and teachers feel comfortable and the overall atmosphere of mutual cooperation among parents, pupils and teaching staff.

3 The Digital Classroom

By equipping a common classroom with the digital technologies we get a “digital classroom”. Not only the traditional blackboard and the desks are there but also a multi-touch sounded interactive whiteboard, teacher’s computer connected to the school network and the Internet, a projector and other necessary accessories – a special software for both, teachers and learners, educational software for teachers and learners, voting device for each student, a projector, camera; or tablets and smart phones.

The highest level of a digital class is a 1:1 learning: each student has his own device (a notebook, netbook or tablet). With the ICT tools the students and teachers can share individualized instructional materials, the results of their work and get the feedback. The teaching process is focused on the student and it can be very individualized, yet there still plays role a classroom atmosphere – still, children are using the blackboard that serves as a mean of presentation and where are the results shown. A home preparation is supported by simple and practical software tools, mobile apps and sharing on-line materials and by communication with the teachers.

Lower forms of digital classes are made of an "inclusive" model of education so called "the digital nests" with several computers available (usually one computer for 3-4 students) for group or project work. There should be an end product of the group work such as presenting the results to the others on the interactive whiteboard. That means a real teamwork among students and their involvement into the educational process. Also, there is a valuable opportunity to use the "digital nest" for an individual work or for testing. This *lower form* is suitable as a good preparation for a switchover to a fully digital classroom. It is possible to use a standard computer classroom with an aim to serve as a common classroom for variety of subjects.

4 Using the ICT in Math lesson

The objective of the lesson was a presentation and practicing of multiplication and division by 10, 100 and 1000; and the revision of millions subtraction. The lesson took place in the 7th grade of Practical Primary School with 10 learners. The following overview shows what activities and technologies were used.

TIME	*TEACHER'S ACTIVITY*	*LEARNER'S ACTIVITY*	*TECHNOLOGIES USED*
2 min	Introduction: main aims of the lesson	Listens, follows the whiteboard	Interactive whiteboard
6 min	Practice: Math: warm up – exercise practicing small and large multiplication tables, student responds by using a voting device Activity - multiplication and division (from the simple to the more complicated), shows the results	Corresponds by using a voting device - writes the results Self-assessment	A voting device A Smart Book exercise Interactive whiteboard
15 min	Presentation followed by practicing:	Listens to the Teacher Activities – filling in	Interactive whiteboard A Smart book

	multiplication and division 100, 1000	the “numerical snake”, a randomly drawn exercise; the student replies	exercise
10 min	Individual work: division and multiplication; sends the file, checks the results, shows the results.	Receives the file, working with a laptop; sends the results back to the teacher	Notebook Special directing and controlling software Interactive whiteboard
10 min	Revision, practicing a million subtraction, deals with a voting device	Uses a voting device, answers the questions, displaying the results, check of the results	Interactive whiteboard A voting device
2 min	Conclusion: a lesson evaluation, home assignment	Makes notes – home assignment	Interactive whiteboard A notebook

Tab. 1 A lesson plan – Math

The Interactive whiteboard is a medium of presentation in the introductory part of the lesson. The pupils follow basic points that are accompanied by the teacher’s speech. While revising the last topic the voting devices are used; pupils answer the questions asked by the teacher; the teacher displays these questions on the Interactive whiteboard.

The students can immediately check their results with the correct ones. The arithmetical problems are randomly repeated. The alternative can be the exercises in Smart Notebook when the learners take turns and use the whiteboard to solve the assignments. A good source of inspiration could be so called Lesson Activity Toolkit, e.g. matching the correct results to a randomly drawn exercises or a game “pelmanism” (to the card with an arithmetical problem must be found the correct result).

The software includes variety of activities that can be changed appropriately to provide learners with original and dynamic educational content.

New topic is presented by using the Interactive whiteboard along with a teacher’s explanation. The lecture is followed with exercises that practiced learners’ knowledge. The teacher can create an exercise using the Lesson Activity Toolkit. An extensive activity consists of monitoring pupil’s work (he is working with laptop to fulfill the task using the educational software or Smart Netbook) and of working with students according to their individual needs. The pitfall of and individual work are differences in learners’ knowledge and skills. Teacher can take advantage of the students who have finished the task by using them to help their classmates.

5 Conclusion

The project brings valuable experiences to the school. It is realized step by step and it is not comparable to the pilot project *21*; it only draws information and concurrent results that can school use to compare its own achieved knowledge and experience.

The teachers are more reserved. They show appreciation and enthusiasm when they meet the "digital class" and they are happy to participate in workshops but when they come into terms with the first problem – which is often a technical issue – they give up saying: "I can not handle it" or "It's very nice but I will keep my teaching style that works". They use an Interactive whiteboard including their own digital textbooks without involving students' laptops or other devices.

The teachers can also define the differences in education that took place some time ago – without use of ICT – and contemporary education with everyday ICT use; mot of them taught in both periods.

They agree that education is more inventive, demonstrative and dynamic, very attractive for children. They think that the biggest advantage are the digital materials – especially because they are easy to store (there is no need for special rooms or storerooms), they can be re-used and updated. Also, interactivity and creativity are higher than during traditional lesson. Another advantage lies in a digitalization of pedagogical documentation. They claim that the biggest disadvantage is a time-consuming preparation and constant monitoring the news and, also, self-improvement; frequent technical difficulties that hamper a trouble-free use.

The Pedagogy science has not inspected, verified and not documented all the pros and cons of using the multimedia by the whole class. This is an ongoing process and it is likely that the educational process will be enriched by something with a little amount of didactic and organizational recommendations.
There exist legitimate experiences from abroad that the process must start with the school management. The school should have a vision that incorporates the ICT into the school environment and that it must become a part of the school culture. Another implicit conditions are the further education for teachers, good infrastructure accompanied by the technical support and extra educational programs, on-line resources and, last but not least, a good practice examples.

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On the Pluricanonical Maps

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Abstract:
One of the applications of Reider's theorem is the classical result of Bombieri to the effect that the linear system $|mK_X|$ is base point free for $m \geq 4$. This implies, in particular, that the base point freeness of pluricanonical maps is determined numerically. We investigate here to what extend the assumptions in the Reider's theorem are optimal and whether the existence of base loci of pluricanonical systems is numerically determined also for small values of m. In order to do this, we consider two linear systems: $|3K_X|$ and $|2K_X|$.

Key words: pluricanonical map, linear system, base point.
MESC: H10

1 Introduction

Here we investigate canonical maps on surfaces of general type, in particular we study base points of pluricanonical maps.

One of the applications of Reider's theorem is the classical result of Bombieri to the effect that the linear system $|mK_X|$ is base point free for $m \geq 4$. This implies, in particular, that the base point freeness of pluricanonical maps is determined numerically. We investigate base loci of $|2K_X|$ and $|3K_X|$.

The first observation is that if $K_x^2 \geq 2$, then we can again apply Reider's theorem to show that $|3K_X|$ is base point free. Hence the interesting case is that of minimal surfaces with $K_x^2 = 1$. One example is a Godeaux surface. But a surface with $K_x^2 = 1$ need not to be a Godeaux surface. We construct another example in the weighted projective space with weights 5, 2, 1, 1.

Then we show that $|2K_X|$ in the later example is base point free. The main conclusion of this note is that the existence of base points for pluricanonical maps is not completely governed by numerical properties.

2 Notations and basic facts

In this paper a surface X will be always a compact, connected 2-dimensional complex manifold.

To begin with, let us recall the definition of a rational map from [6, I.3].

Definition 1.1 *Let X, Y be varieties. A rational map $\varphi : X \to Y$ is an equivalence class of pairs (U, φ_U), where U is a nonempty open subset of X, φ_U is a morphism from U to Y, and where (U, φ_U) and (V, φ_V), are equivalent if φ_U and φ_V agree on $U \cap V$.*

Example 1.2 *A rational map defined by global sections of a line bundle L. Let $s_0, \ldots, s_N$ be a basis of $H^0(L)$. Then the mapping*

$$\varphi_{|L|} : X \ni x \longrightarrow (s_0(x) : \cdots : s_N(x)) \in \mathbb{P}^N$$

is well defined away of the common zero locus of all sections of L, i.e. away of the base points of the linear system |L|.
An invertible map in this category is called a birational map.

Let *Div(X)* be the group of Cartier divisors on X and *Pic(X)* be the group of line bundles on X, i.e. *Div(X)* modulo the linear equivalence.

We denote by K_X the **canonical divisor** of X. It is the element of *Div(X)*, such that $O_X(K_X) = \Lambda^2 \Omega_X^1$, where Ω_X is the sheaf of one forms on X.

Definition 1.3 *Let L be a line bundle (or a divisor) on X. L is said to be* ***base point free****, if for every point $x \in X$ the restriction map*

$$H^0(X; L) \to H^0(x; L_x) = C$$

is surjective. Here L_x denotes the skyscraper sheaf over x.

In particular the rational map defined by global sections of a line bundle is a regular map if the line bundle L is base point free.

We say that L is **semiample**, if lL is base point free for l >> 0.

In [6, II.8.18] we can find

Theorem 1.4 *(Bertini's Theorem)*

Let X be a nonsingular closed subvariety of P_k^n, where k is an algebraically closed field. Then there exists a hyperplane $H \subseteq P_k^n$ not containing X, and such that the scheme $H \cap X$ is regular at every point. Furthermore, the set of hyperplanes with this property forms an open dense subset of the complete linear system |H|, considered as a projective space.

Definition 1.5 A divisor D is said to be **nef**, if

$$D.C \geq 0,$$

for every curve $C \subset X$.

For any line bundle L on X we denote by

$$h^i(L) = \dim H^i(X, L)$$

the dimension of the cohomology groups of L and we denote by

$$\chi(L) = h^0(L) - h^1(L) + h^2(L)$$

the Euler-Poincar´ characteristic of L.

Similarly, for a divisor D, we write

$$h^i(D) = h^i(O(D)).$$

All n-dimensional compact, connected complex manifolds X can be classified according to their Kodaira dimension, denoted by κ(X).

Definition 1.6 Let X be a smooth compact complex manifold of dimension n.

The **Kodaira dimension**, κ(X), of X is defined to be

$$\kappa(X)=\begin{cases}-\infty, & \text{if } H^0(X,\mathcal{O}_X(mK_X))=0 \text{ for all } m\in\mathbb{N},\\ (tr.deg_{\mathbb{C}}\bigoplus_m H^0(X,\mathcal{O}_X(mK_X)))-1, & \text{if } H^0(X,\mathcal{O}_X(mK_X))\neq 0 \text{ for some } m\in\mathbb{N}.\end{cases}$$

Theorem 1.7 *(Adjunction formula)*

If Y is a complex submanifold of codimension 1 of the complex manifold X, then

$$K_Y = K_X$$ Ä $$O_X (Y) |Y.$$

Lemma 1.8 *[2, Lemma VIII.9] Let S* Ì P^n *be a d-dimensional complete intersection. Then* $H^i (S, O_S) = 0$ *for* $0 < i < d$*.*

Let D Î *Div(X)* be a divisor on X.

Theorem 1.9 *(Riemann-Roch Theorem)[1, Theorem 1.3.1]*

$$\chi(O_X (D)) = \chi(O_X) + \frac{1}{2} D.(D - K_X).$$

Remark 1.10 *(Serre duality)[2, Remark I.13.(i)]*

Serre duality gives

$$h^i (D) = h^{2-i} (K_X - D), \text{ for } 0 \le i \le 2.$$

3 Canonical maps on surfaces of general type

A surface of general type is an algebraic surface X with $\kappa(X) = 2$. Most surfaces are in this class. We have a pluricanonical map

$$\varphi m = \varphi|mKX| : X \dashrightarrow K\, P^{Nm} \ (N_m = h^0(mK_X) - 1)$$

The following two theorems are relevant in this section, see [3].

Theorem 2.1 *(Bombieri)*

Let X be a minimal surface of general type. Then

i) if $m \ge 4$*, then* φ_m *is base point free;*

ii) if $m \ge 5$*, then* φ_m *is an immersion except for (−2)-curves which get contracted to the nodes.*

It is not hard to show, that the theorem of Bombieri follows from Reider's Theorem [7] (which is a modern approach).

Theorem 2.2 *(Reider)*

Let X be a surface and let L be a nef line bundle. Then

i) If L^2 ³ *5 and P is a base point of the linear system* $|K_X + L|$*, then there exists an effective divisor D through P with*

a) $D.L = 0, D^2 = -1$ *or*

b) $D.L = 1, D^2 = 0.$

ii) If $L^2 \ge 10$ *and two (possibly infinitely near) points P and Q are not separated by* $|K_X + L|$*, then there is an effective divisor D containing P and Q with*

a) $D.L = 0, D^2 = -2$ *or* -1*, or*

b) $D.L = 1, D^2 = -1$ *or 0, or*

c) $D.L = 2, D^2 = 0.$

It is natural to wonder to what extend the result of Theorem 2.1 is optimal. To begin with, we ask if the tricanonical system $3K_X$ might fail to be base point free, and if so if one can derive more properties of the surface X, if this happens. The first observation is that if $K_x^2 \ge 2$, then we can again apply Reider's theorem to show that $3K_X$ is base point free. Hence the interesting case is that of minimal surfaces with $K_x^2 = 1$.

3.1 Minimal surfaces of general type with $K_x^2 = 1$.

We will use to our construction the weighted projective space. Weighted projective spaces are generalization of the projective spaces. They are usually singular. Here, we are interested only in the special case of P := P(5; 2; 1; 1) and we refer to [5, I.5] for general theory.

According to [5, Proposition I.5.15] P is singular at

$$P_1 = [1 : 0 : 0 : 0] \text{ and } P_2 = [0 : 1 : 0 : 0]:$$

Let f be a general polynomial of weighted degree 10. Then f does not vanish at P_1, nor at P_2 and defnes a smooth surface X_f^1 in P.

For example

$$f = x^2 + y^5 + z^{10} + t^{10} Î \; 2\, C[x; y; z; t]$$

has this property.

By [5, Proposition I.6.8] and Theorem 1.7

$$K_X \; @O_P(1);$$

hence X is a surface of general type with $K_x^2 = 1$ and $p_g(X) = 2$.

The canonical pencil $|K_X|$ has exactly one base point P.

3.2 Base points of multicanonical map

We consider example of a minimal surface of general type with $K_x^2 = 1$.

Theorem 2.3 *Let X be a general surface of (weighted) degree 10 in P(5; 2; 1; 1). Then*

a) $|3K_X|$ has a base point P ,

b) $|2K_X|$ has no base points.

Proof. We know from Bertini's theorem 1 that generic member of $|K_X|$ is smooth away of P. The equality $K_x^2 = 1$ implies that all elements of $|K_X|$ are smooth at P . Hence there exists a smooth curve C in $|K_X|$. By Adjunction

$$g(C) = 1 + \frac{1}{2}(K_X + C).C = 2.$$

Moreover

$$K_C = K_X + C|_C = 2K_X|_C \tag{1}$$

and

$$K_X|_C = O_C(P).$$

In order to prove a) we consider the sequence of sheaves

$$0 \to O_C(3K_X|_C - P) \to O_C(3K_X|_C) \to O_P(3K_X|_P) \to 0$$

which is equivalent to

$$0 \to O_C(2P) \to O_C(3P) \to C_P \to 0,$$

where C_P is the skyscraper sheaf at P .

We have the associated long exact cohomology sequence

$$0 \longrightarrow H^0(\mathcal{O}_C(2P)) \longrightarrow H^0(\mathcal{O}_C(3P)) \xrightarrow{\alpha} \mathbb{C} \longrightarrow H^1(\mathcal{O}_C(2P)) \longrightarrow H^1(\mathcal{O}_C(3P)) \longrightarrow 0.$$

Now, we have

$$h^1(O_C(3P)) = h^0(K_C - 3P) = 0$$

by Serre duality 1.10 and $\deg(K_C - 3P) = -1 < 0$.

On the other hand

$$h^1(O_C(2P)) = h^0(K_C - 2P) = h^0(K_X + C|_C - 2P) = h^0(O_C) = 1$$

again by Serre duality and (1).

This shows that the map α is not surjective, which means that P is a base point of $|3K_X|$ as asserted.

For claim b), we observe that $K_X = O_P(1)$, so that y gives a section which does not vanish at P. Indeed, otherwise it would be *y, z* and *t* vanishing at P, which implies

$$P = [1 : 0 : 0 : 0] = P_1 ,$$

a contradiction. Hence P is not a base point of $|2K_X|$. Since the only base point of $|K_X|$ is P , this implies that $|2K_X|$ is base point free.

4 Conclusion

In examples considered here, the tricanonical map has base points. We were hoping to find examples of minimal surfaces of general type with $K_x^2 = 1$ such that $3K_X$ has no base points. It seems that no such examples are known. We hope to come back to this problem in the future work.

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INQUIRY-BASED MATHEMATICS EDUCATION – A CHALLANGEAND A CHANCE

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Abstract:
In the article we present inquiry-based mathematics education as a method with a potential to achieve positive changes in education of mathematics and of natural sciences. We inform about some activities done within the Fibonacci project and include examples of presentation of several tasks, which create environment suitable for use of inquiry-based methods in primary school education context.
Keywords: Inquiry-based mathematics education, problem, solving tasks in mathematics.
MESC: C70, D40

1 Introduction

A complex comparative study into the state of education of natural sciences in EU countries[1] has shown the decreasing level of knowledge of mathematics and natural sciences among pupils. It has also attempted to describe main problems and formulate suggestions necessary to enable the desired positive changes. One of the suggestions was to innovate education methods by means of introduction of inquiry based ways of working with pupils and education of teachers so that they could effectively use such methods. Simultaneously, creating and developing nets of teachers and their colleagues from universities, research institutions and practice should be encouraged. In our contribution we recall some basic features of a method called inquiry-based mathematics education, include some experience from the Fibonacci project, the project team of which the authors were members, and present two problems, which create environment suitable for applying the inquiry-based techniques in the elementary school context.

2 Inquiry-based education

The inquiry-based education is often characterized as a constructivist way of education and teaching (inquiry is meant not only as research but also as truth-seeking).[2] In the last decade it has become increasingly popular as a label for positive changes, especially in mathematics and natural sciences education. It has generated great expectations, even though at the other hand there are some doubts whether “*this concept denotes something indeed new in the processes of teaching and*

[1] Report for The Nuffield Foundation; Osborne a Dillon Science Education in Europe: Critical reflections, 2008
[2] „Inquiry“ is defined in various ways in literature. The definition used in *Linna, Davise, Bella, 2004, p. 15*, taken over in *T. Janík* and *I. Stuchlíková (2010, p. 21)* states that: „Inquiry is a purposive process of formulating problems, critical experimenting, weighing alternatives, planning, investigating and verifying, drawing conclusions, searching for information, making models of studied phenomena, discussion and forming coherent arguments. “

learning or only in other words emphasises aspects of something that the pedagogical practice has de facto done for long" *(Janík, Stuchlíková, 2010, p. 21).*

Already J. Vyšín in his classical work Kapitoly o problémovém vyučování matematice defines the concept of a mathematical problem, the main feature of which is "seeking the method of solution, a kind of research work at a small scale" (1977, p. 69). The Socratic dialog may be regarded as a prototypical way of inquiry. In more recent educational methodologies this place is occupied by application of heuristic methods or problem solving. From the point of view of didactics of mathematics we recall that inquiry typically incorporates experimental techniques which develop reasoning and instrumental skills of pupils is an important feature of learning by doing of A. Z. Krygowské (1977) and H. Siwek (2005), theory of problem education in the concept of G. Polyi (1973), scheme of solving a problem task suggested by M. Zelina (2000), which is known as DITOR, or a very inspiring research of J. Kopka (1999, 2007). In our contribution we regard both approaches – inquiry-based education and problem education – as in fact equally relevant. Our concept could be regarded as closest to "discovery through problem solving" (Novotná, 2000, 2004).

The above presented methods are mirrored in the current elementary school curriculum: it is especially the thematic area "nonstandard application tasks and problems" penetrating the whole education content that gives space for inquiry-based education. Pupils have to solve mathematical problems, inquire, reason and discover *the method of solution* because their experience is of no help. Also other authors such as *J. Kopka (2007), O. Šedivý, J. Fulier (2004)* point out that nonstandard tasks lead to discoveries, finding new ways of solution and by this develop cognitive skills of pupils. These features make such tasks suitable for work with talented pupils (*Malinová, 2013*).

3 Fibonacci project

The Fibonacci project is a European project which reflects the agreement of international scientific community on the importance of inquiry-based science and mathematics education (IBSME).[3] It was realised between 2010 a 2013 with approx. 60 university and non-university institutions, 3.000 teachers and more than 45.000 pupils and students. It was one of four great international projects of the 7th European Framework Programme, which dealt with teaching mathematics and natural sciences. Apart from research these projects also aimed at implementing and spreading their results into elementary and secondary school contexts.

The project gave rise to *twin centres* which provide teachers with professional support on inquiry-based education, serve as meeting points and a means of communication, visits in classes, etc. They also support teaching materials and their help in sharing them. One of the twin centres – based on the Faculty of Education, University of South Bohemia in České Budějovice[4] – offered its co-operation to Department of mathematics, Faculty of Education, Palacký University in Olomouc in the form of joint seminars for teachers and of other pedagogical activities in presentation of inquiry-based education.

[3] http://www.fibonacci-project.eu
[4] http://www.pf.jcu.cz/stru/katedry/m/fibo.html

In our contribution we include two examples which suggest ways of application of inquiry-based education. They were carried out when one of the authors, A. Fleková, taught mathematics in the 7th grade of elementary school.

4 Example 1 – Euler characteristic

Activity description, instruments

In the example we inquire the ways of determining the number of vertices, edges and sides of convex polyhedrons.[5] The concept may be used already in experiments in elementary school geometry[6] or during discovery and analysis of features of solids in elementary school stereometry.[7] Modern instruments such as Polydron™, Geomag™ or Magformers™, which enable the user to build convex polyhedrons, are suitable. In our example Polydron™ has been used.

Assumed knowledge:

Usual solids (convex polyhedrons) – name, the concepts of vertex, edge, side.

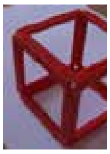
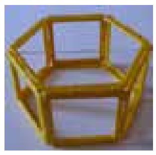
Stages of discovery:

- *Task for students:*

Create models of tetrahedron, cube, regular square pyramid, regular five-sided pyramid, and regular six-sided prism. Look for the relation between number of sides edges and vertices in each polyhedron.

- *Experiment realisation and its recording (pupils do themselves based on teacher instructions):*

Write the number of vertices (v), sides (s) and edges (h) of each polyhedron in a table and try to find out the relation between v, s and h.

number of	**tetrahedron**	**cube**	**regular square pyramid**	**regular five-sided pyramid**	**regular six-sided prism**
vertices (v)	4	8	5	6	12
sides (s)	4	6	5	6	8
v + s	8	14	10	12	20
edges (h)	6	12	8	10	18

Tab. 1 Number of vertices, sides and edges of convex polyhedrons

[5] Kopka, J. *Výzkumný přístup při vyučování matematice*. Ústí n.L.: UJEP 2004
[6] Molnár, J. aj: *Matematika pro 5. ročník ZŠ.* Olomouc: Prodos 1998, 2. díl, pp. 29-30
[7] http://home.pf.jcu.cz/~math4all/aktivity_u_s.php?stupen=2_zs&sekce=18&akt=4

· *Inquiry – discovery (with teachers' help):*

From the data in the table we have found out that

- Ø The greatest number for all solids is the number of edges. Furthermore, it is obvious that
- Ø If we make the sum of two smaller numbers on each row (in all cases $v + s$), we get a number which is only slightly different from h, the number of edges. How slightly? Is the difference the same for each row?
- Ø After a closer look we find out that there holds $v + s = h + 2$

· *Hypothesis formulated – by pupils or with teachers' help*

- Ø In an arbitrary convex polyhedron with v vertices, s sides and h edges there holds v + s = h + 2 or v – h + s = 2.

· *Hypothesis verification:*

Make models of some other solids such as cuboid or octahedron and verify the hypothesis: does the above assumption hold for these solids as well?

Methodological reflection of the activity realisation:

We believe that the activity can be done with pupils only after they had learned how to use Polydron™, i.e. after they had learned to assemble the solids. This enables the pupils to fully concentrate on the specific didactic aim during the teacher controlled activity. The tested pupils had no problems in assembling the solids and in using the necessary terminology (vertex, edge, side). They filled in the table according to teacher's instructions. However, discovering the relation was more difficult. Here, the teacher's aid was necessary in questions such as "*How many edges are there in a cube? What is the sum of vertices and sides? What is the difference between the two numbers? What about other solids? Is this difference always the same? Does this imply anything?*" After this all students were able to find the correct answer. The verification stage, i.e. assembling models of other polyhedrons and finding out whether the relation is valid in them too, was very popular and interesting for the pupils. After the manipulative stage had finished and the relation had been discovered, pupils commented that the activity was interesting and entertaining and let them not only practice concepts they had already met but also discover some new connections.

5 Example 2 – Way to school (Walking in the city)

Activity description, instruments

This example is usually referred to as *paths in a square net*[8] in literature. We examine determining the number of paths which can be constructed in a square net in order to get from point A to point B. When doing this activity with pupils it is recommended to use a worksheet with several identical square nets, in which the pupils can draw the paths (experiment).

[8] Kopka, J.: *Hrozny problémů ve školské matematice*. Ústí n. L.: UJEP, 1999
Kirkby, D.: *Investigation Bank Book 5, 7, 17*. Sheffield: Dickens & Son, 1986
Engel, T., Varga, T.,Walser, W.: *Zufall oder Strategie?* Stuttgart: Ernst Klett Verlag, 1974

Assumed knowledge:

No special mathematical knowledge is required; the ability to draw in a square net and to make records of individual tries by using suitable symbols (arrows, numbers, letters) is assumed.

Stages of discovery:

- *Task for students:*

Figure 1 is a city plan. Adam (A) can use different paths to get to school (S). However, he is allowed to move forward only. He is not allowed to go back. In the context of the plan this means that he can go only up (north), denoted by *s*, or right (east), denoted by *v*, i.e. only in directions of arrows ↑ or →. How many different paths can he take to get to school?

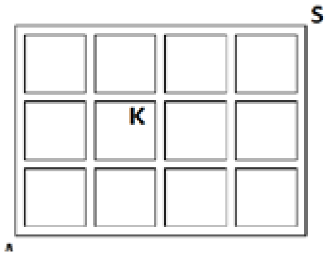


Fig. 1 The city plan

- *Experiment realisation and its recording (pupils do themselves based on teacher instructions):*

Pupils work with a worksheet according to teacher's instructions.

Ø On the first square net there is denoted a crossroads (K), which Adam passes on his way to schools. Using colour pencils draw all possible ways he can get to the crossroads and find out how many sections he has to take in each case. By a section we mean a part of a path between two closest crossroads.

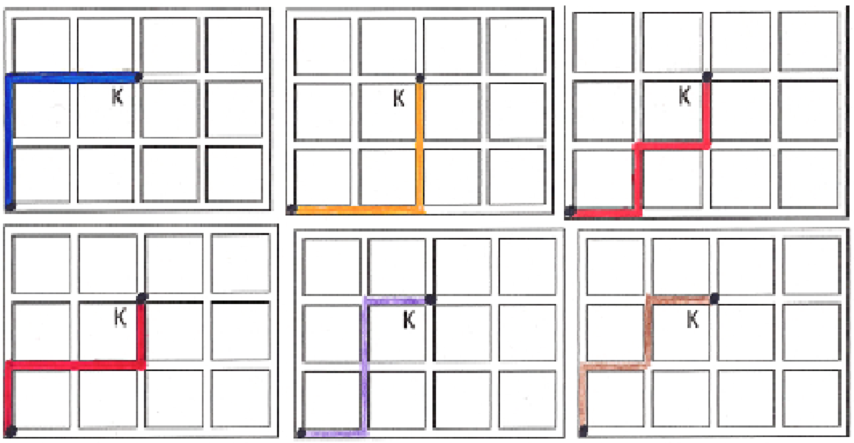


Fig. 2 Various paths

Ø We have found out that the number of paths leading to the crossroads K is 6 and that every such path has 4 sections.

Ø There is only one possible way one can get to each crossroads to the north or to the east of A. What is the number of paths one can take to get to every remaining crossroads in the city plan?

- *Hypothesis formulated – by pupils or with teachers' help*

The number of paths to every next crossroads can be determined as a sum of number of paths leading to the respective crossroads. Figure 3 shows the number of paths leading to every crossroads.

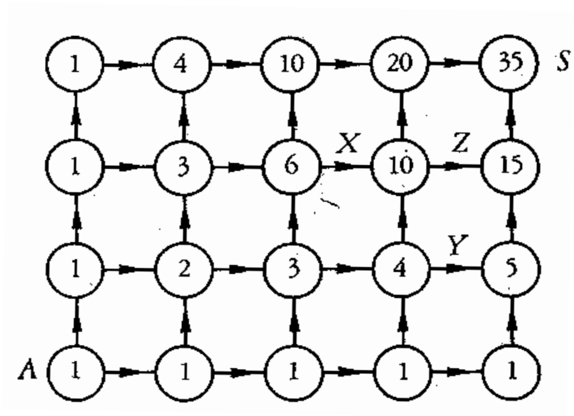


Fig. 3 Numbering the crossroads

- *Hypothesis verification:*

The crossroads 2 can be reached using two paths (1+1), crossroads 3 can be reached using three paths (2+1), etc. Schools S can be reached by 35 different paths, each 7 sections long.

- *Further uses of the idea – suggestions for further discoveries (cf. Kopka, 1999):*
  - Ø Denote the paths to crossroads K by different colours and their sections by arrows, e.g blue path →→↑↑, or use letters such as vvss.
  - Ø Use arrows or letters to denote all 6 possibilities. How will you prove that no more paths exist?

→→↑↑	↑↑→→	*vvss*	*ssvv*
→↑→↑	↑→↑→	*vsvs*	*svsv*
→↑↑→	↑→→↑	*vssv*	*svvs*

Methodological reflection of the activity realisation:

First of all, pupils were told the rules of the activity so that they new directions they could use. Than city plans prepared by the teacher were handed in. After this the pupils started to look for various paths to the first crossroads K. Several pupils guessed the number (instead of finding it) while some others used forbidden directions. After the teacher pointed out these mistakes, most pupils were able to find all 6 paths while respecting the rules.

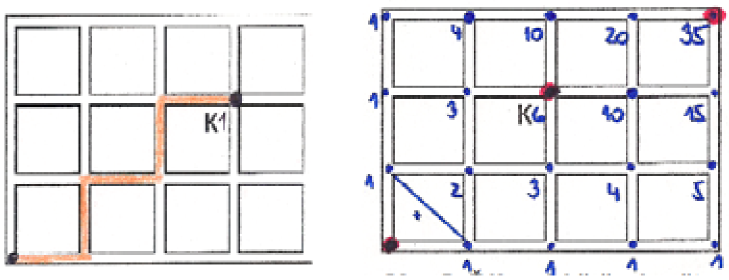


Fig. 4 City plans for advanced pupils

[9] Engel, T., Varga, T.,Walser, W.: *Zufall oder Strategie?* Stuttgart: Ernst Klett Verlag, 1974, p. 15

Faster pupils were given another city plan with crossroads K1. They had to repeat the task, i.e. to find paths from A to K1 using different colours. When looking for paths in this more advanced plan, pupils soon started to ask whether there existed an "easier" way of finding them other than colouring the plan.

The teacher sketched a city plan on board and performed with pupils a step-by-step deduction on how many paths lead to respective crossroads (a sum of paths to previous crossroads). Having done this, the pupils were able to discover the number of ways Adam can take to school.

There were great differences between pupils in this activity. Some of them regarded drawing paths as amusing but did not like counting them, and easily lost their attention and willingness to perform the activity. Some others, on the other hand, enjoyed the activity and started to draw and count the paths themselves. These different attitudes were then reflected in the discussion which followed the activity.

6 Conclusion

In our contribution we have discussed our experience obtained in a specific educational reality concerning two discovery activities. The class, in which they were performed, has average results. The approach we have discussed has been applied on it neither systematically nor on long term basis. In spite of this we believe we can attempt to formulate some (albeit subjective) conclusions.

The *professional competence of teachers* is one of decisive factors in the inquiry-based education. This is especially true for *didactic competence in the specialization* which is a core of professional competence of teachers, "which is the aspect in which teachers differ from other professionals and which makes them irreplaceable by other professionals. These include especially knowledge of the specialization and its didactic approaches, knowledge of the curriculum and of its application as well as the art of professional response to pupils' presentation in class and the ability to make use of this in the educational process" *(Tichá, 2012, p. 25).* However, competences of a *more general nature*, with impact in more than one (school) subject are equally important. These include the competence to motivate pupils, to communicate with them and to use adequate means in evaluating pupils' performance. In our case, this was especially the case of competence of verbal communication of the teacher towards pupils, i.e. accurate wording of questions leading to achieving the task.

We have clearly seen some typical problems connected to introducing inquiry-based approaches. From the point of view of pupils these include inadequate level of motivation and problematic background of insufficient mathematical knowledge and skills. From the point of view of teachers these include (on top of the already mentioned ones) the fact that preparation of such activities is, in a typical educational context, both time- and material consuming. Even though the teacher had prepared a well staged scenario she was not able to make the actual performance of the activity follow it. This implies that the teacher must use their full potential of creativity and flexibility so that even such situations could be used in accordance with principles of inquiry-based education.

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EXAMPLES FROM HISTORICAL MATHEMATICAL TEXTBOOKS WITH USING GEOGEBRA

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Abstract: There was at Italian secondary school named Liceo Scientifico Isacco Newton - Roma successful project "La Nuova Geometria del Compasso Le costruzioni di Lorenzo Mascheroni utilizzando il software GeoGebra". Students F. Fabrizi and P. Pennestrì prepared examples from historical mathematical book Lorenzo Mascheroni: La Nuova Geometria del Compasso presentation in GeoGebra. In our contribution we would like to show how is possible to use GeoGebra in some other cases of historical - mathematical textbooks.

Key words: Franz Močnik, Václav Posejpal, Geometry, method of generating problems
MESC: Secondary, C70, A30, U70

1 Introduction

The idea of establishment of an institute for education and formation of Slovak qualified teachers for village schools arose in the minds of the Slovak enlighteningly thinking and acting followers of Anton Bernolák (Bernolaks) in Spišská Kapitula (see Gejdoš (2007)). The educated Spiš bishop Ján Ladislav Pyrker (1772 – 1847) supported Bernolaks. In 1819 he made a journey throughout his diocese together with the Secretary Dr. Ladislav Zábojský and the Bernolak writer Juraj Páleš. They all focused on the schools of the Spiš diocese. The bishop found the schools in bad conditions, so he decided to establish a pedagogical institute for education of village teachers. In 1918 Juraj Páleš became the first director of the Pedagogical Institute in Spišská Kapitula. It was aimed to produce qualified teachers for village schools. In his decree it is stated: "Relying on the outstanding willingness of Your Excellence, by which you sacrifice your abilities and efforts as much as you can, I entrust the administration of the Institute for preparation of teachers, that is to be opened on November 10th, to Your hands."[1]

The State archive in Levoča has Archive Fund of the Teachers Academy in Spišská Kapitula. This fund contains some documents from Teachers Institute. Another written relic following the Teachers Institute is the Archive Fund of the Teachers academy in Spišská Kapitula and the Library of Teachers Academy in Spišská Kapitula, which are placed in Bishop Archive of Spiš in Spišské Podhradie. Both of the funds were established by the delimitation of torsoral documents from the Bishop Archive Fund of Spiš, by collection activities organized by diocese of Spiš on the occasion of the 180th anniversary of the Teacher Institute foundation. In these funds it is possible to find some mathematical textbooks, which contain interesting examples. These examples will be

[1] In the State Archives in Levoča in the collection of the Office of the Bishop in Spišská Kapitula there are two letters by J. L. Pyrker to J. Páleš.

illustrated by the open source software GeoGebra. We choose examples, which are suitable for the secondary schools or future Maths teachers.

2 Some examples from Jaroslav Havelka textbook

First part of examples we will demonstrate from the textbook of Jaroslav Havelka: *Geometria pre ústavy učiteľské (Geometry for Teachers´ Institutes).* This textbook is from Library of Teachers Academy in Spišská Kapitula and we have Slovak translations from this textbook, which was made by Vladimír Hapala (see Havelka (1924)).

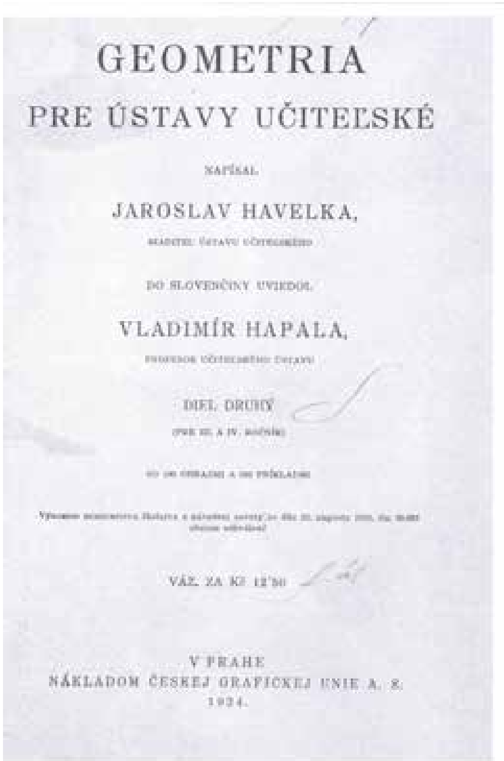
GEOMETRIA
PRE ÚSTAVY UČITEĽSKÉ
NAPÍSAL
JAROSLAV HAVELKA,
DO SLOVENČINY UVIEDOL
VLADIMÍR HAPALA,
DIEL DRUHÝ
VÁZ. ZA Kč 12'50
V PRAHE
NÁKLADOM ČESKEJ GRAFICKEJ UNIE A. S.
1924.

Fig. 1

We choose the following example: Proof, that equilateral triangle inscribed the circle has the same area than the half of the equilateral hexagon inscribed the same circle.

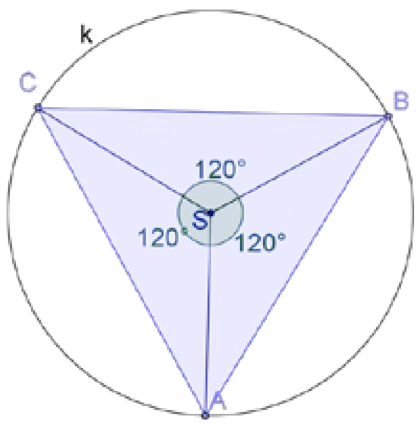

Fig. 2

Solution: If we have the circle with the centre S and radius r, we can use for the equilateral triangle *ABC* inscribed this circle, following expression for the area of triangle *ABS* (see Figure 2):

$$S_1 = \frac{1}{2}r^2 \cdot \sin 120° = \frac{1}{2}r^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2$$

The area S of the whole triangle is

$$S_1 = 3S_1 = 3 \cdot \frac{\sqrt{3}}{4}r^2 = \frac{3\sqrt{3}}{4}r^2.$$

If we have one half of the equilateral hexagon - quadrilateral *KLMN* (see Figure 3), we can make the same algorithm such by triangle *ABC*.

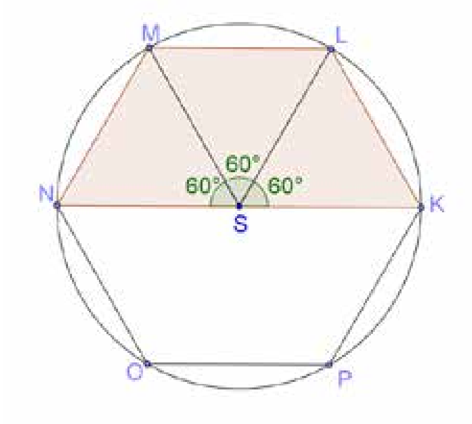

Fig. 3

The area of triangle *KLS* is:

$$S_2 = \frac{1}{2}r^2 \cdot \sin 60° = \frac{1}{2}r^2 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2$$

The area S_3 of the whole quadrilateral *KLMN* is

$$S_3 = 3S_2 = 3 \cdot \frac{\sqrt{3}}{4}r^2 = \frac{3\sqrt{3}}{4}r^2.$$

We became, that $S = S_3$.

This example was possible to solve in another ways. Both figures - triangle *ABC* and quadrilateral *KLMN* contain from three same triangles (*ABS*, *KLS*). For this reason it is enough to compare the areas S_1 and S_2. We can use now that $\sin 60° = \sin 120°$ and

$$\frac{1}{2}r^2 \cdot \sin 60° = \frac{1}{2}r^2 \cdot \sin 120°$$

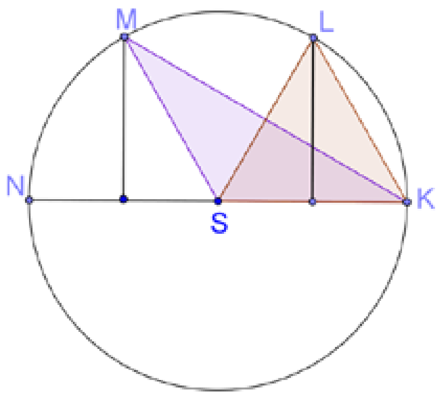

Fig. 4

We can use also that the triangles *ABS*, *KLS* have the sides *AS*, *KS* with the same length *r* and height also with the same length. We can represent these two triangles in the same picture, because the triangles *ABS* and *KSM* are the same (see Figure 4). We can demonstrate that the areas of both figures - triangle *ABC* and quadrilateral *KLMN* have the same area.

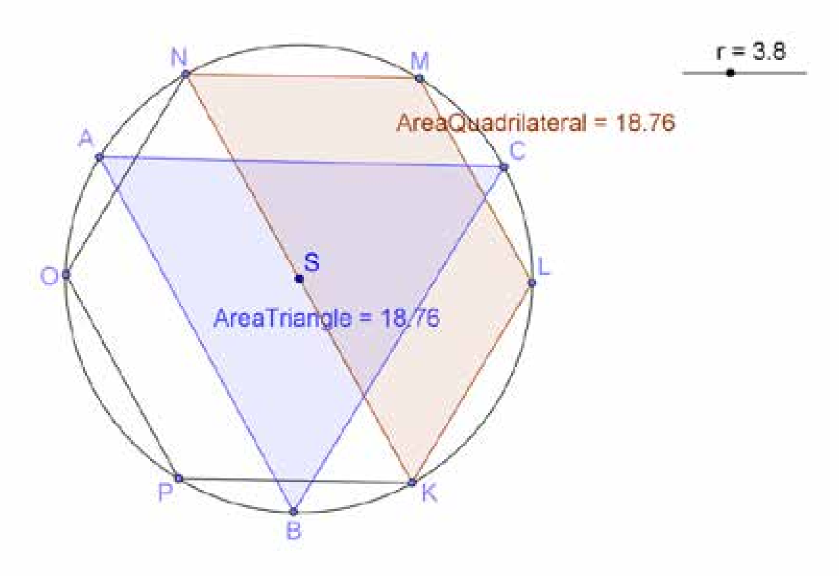

Fig. 5

Pupils have sometimes wrong interpretation of the Figure 5, that this is proof. We can show them cases in which are possible to see the picture prepared by computer programme has wrong information.

According Gunčaga, Fulier, Eisenmann (2008) we can solve following example:

Find the local minimum of the function

$$y = x^4 - \frac{1}{5}x^2 + 1.$$

We can prepare different Figures by GeoGebra:

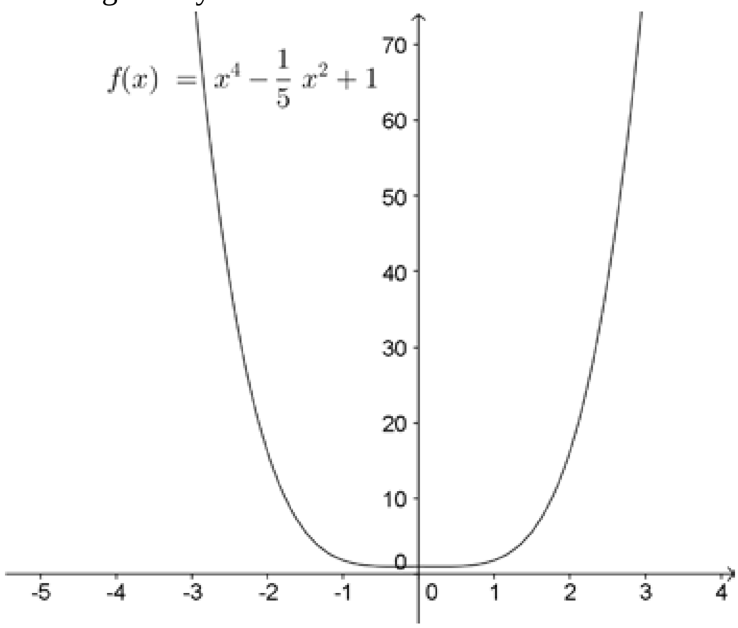

Fig. 6

Pupils give from first figure wrong meaning that the function f has one minimum at point 0.

The second figure show that in reality the function has two local minimums and there exists at point 0 local maximum.

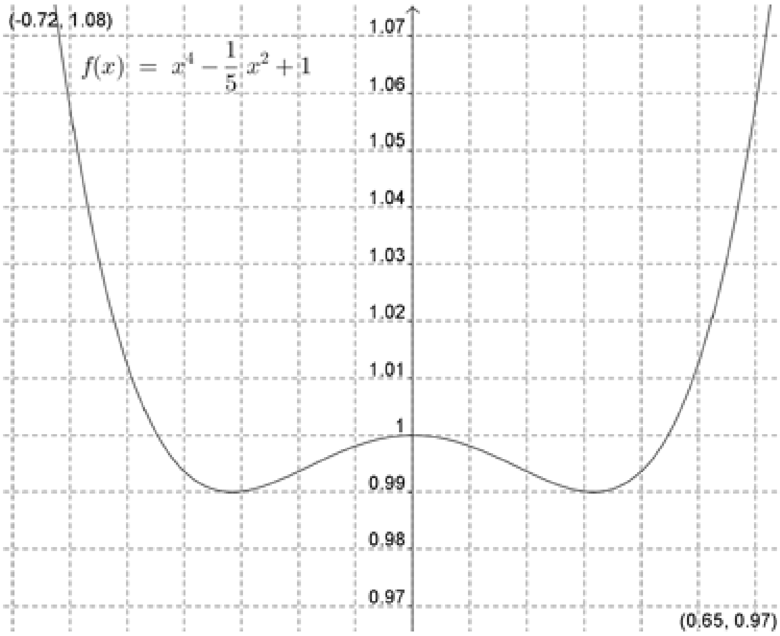

Fig. 7

The correct proof will be realized with help of the differential calculus (first and second derivative).

3 Some examples from Václav Posejpal textbook

Second part of examples we will demonstrate from the textbook Václav Posejpal: *Aritmetika pre ústavy učiteľské (Arithmetics for Teachers´ Institutes).*

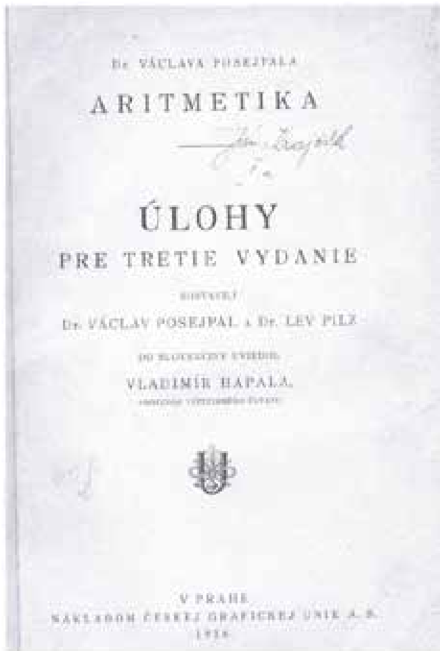
Dr. VÁCLAVA POSEJPALA
ARITMETIKA
ÚLOHY
PRE TRETIE VYDANIE
Dr. VÁCLAV POSEJPAL a Dr. LEV PILZ
VLADIMÍR HAPALA
V PRAHE
NÁKLADOM ČESKEJ GRAFICKEJ UNIE A. S.

Fig. 8

This textbook is also from Library of Teachers Academy in Spišská Kapitula and we have Slovak translations from this textbook, which was also made by Vladimír Hapala. We use exercise book from this textbook (see Posejpal, Pilz (1926)).

The following example shows that the data in some examples were used from real life in the Czechoslovak republic between two World wars. The example is about calculating of percentages different parts of agriculture earth for different commodity: corn, barley, potatoes, oat, rye, and sugar-beet. The data are from different parts of the country: Slovakia, Bohemia, Moravia, Silesia and Carpatho-Ukraine. This example is possible to use also by Science Education (see Rochovská (2011)).

64

27. Dľa skúmania r. 1920 bolo osiaté v Československej republike:

	v Čechách *ha*	na Moravě *ha*	ve Slezsku *ha*	na Slovensku *ha*	v Podkarpatské Rusi *ha*
z ornej pôdy:	2,463.000	1,151.699	204.440	1,917.897	229,808
1. pšenicou ozim.	183.796	99.251	11.814	252.579	23.045
jar.	44.976	9.764	1.457	9.496	519
2. žitom oz.	456.532	182.105	31.176	196.490	17.575
jar.	10.182	5.769	1.122	3.304	1.514
3. Jačmen oz.	2.591	1.999	425	4.987	1.082
jar.	215.229	130.892	20.645	311.986	3.805
4. ovsom	379.005	153.597	36.123	203.629	29.552
5. zemiaky ran.	5.629	2.257	1.061	7.152	865
pozdné	230.802	129.769	21.607	177.397	30.233
6. cukrovkou	11.265	59.440	3.161	36.011	52

a) Vypočítajte, koľko % ornej pôdy každá z uvedených plodín zaberala v jednotlivých zemiach a potom v celej republike!

b) Keď meria plocha celej republiky 140.480 km^2, koľko % pripadá na každú plodinu a koľko na všetky dohromady?

c) Koľko % ornej pôdy pripadá v jednotlivých zemiach na ostatné plodiny?

d) Koľko % celého územia našej republiky tvorí orná pôda?

Fig. 9

We solve now following quadratic equation with real parameter b from this textbook:

$$3x^2 - (b-9)x - 3b = 0 \quad (1)$$

Discriminant of this equation is $D = (b-9)^2 + 36b = b^2 - 18b + 81 + 36b =$

$$= b^2 + 18b + 81 = (b+9)^2.$$

If $b \neq -9$, then $D > 0$ and equation (1) has two solutions:

$$x_{1,2} = \frac{(b-9) \pm |b+9|}{6}.$$

Hence

$$x_1 = \frac{b}{3}, x_2 = -3.$$

If $b = -9$, then $D = 0$ and equation (1) has one solutions: $x = -3$.

This equation has also geometrical interpretation. Every parabola

$$y = 3x^2 - (b-9)x - 3b$$

must obtain the point [-3,0].

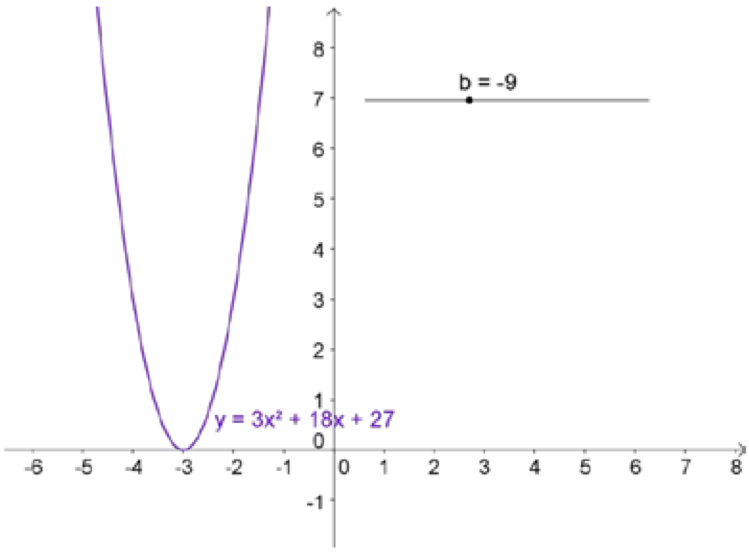

Fig. 10

4 Conclusion

We presented in our article some examples from historical textbooks, which are used in the first Teachers Institute in Spišská Kapitula. This kind of research is possible to realize by every school subject (see Gábor (2012), Kopáčová (2013)). These examples are for pupils in secondary schools and it is possible to illustrate with open source software GeoGebra. This software is only supporting tool (see also Círus (2006), Partová (2011)). For this reason it is important to show pupils contra examples. Other examples those are suitable to show for pupils is possible to find in articles written by Koreňová (2010), Krech (2000) and Konečná (2005).

There exist new topics, which are defined in new Slovak curriculum ISCED3 for secondary schools and which define financial Mathematics and historical interdisciplinary topics for teaching. Materials from historical textbooks are inspirative and motivational support for nowadays Maths teachers.

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DEVELOPMENT OF LOGICAL THINKING USING MATHEMATICAL GAMES

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Abstract:
In this work I have focused not only on the relationship between ontogenesis and phylogenesis in the field of logic, but mainly on the quantitative research combined with the qualitative one.
One of the targets of the work is to carry out, in the form of pedagogical research, the analysis of the present level of logical thinking with pupils of primary and secondary schools. We can presuppose that similarly to the intelligence of an individual, the logical thinking will show relative stability, though some researches mention the possibility of positive influence on the ability of abstraction and logical reasoning (Lee[1], 1990). Another aim is then the answer to the question whether a positive shift is possible in the fields focused on the ability of abstraction and the ability of reasoning through the use of mathematical and logical games.
Key words: logical thinking, mathematical Games
MESC: U60, C70, D40

1 Introduction

The logic can be approached in two ways. Within the first (psychological) approach, the logic deals with thinking processes leading an individual to particular conclusions. It admits that the person makes use of common language, while the concequences are also inferred on the basis of one´s own experience. Thus the disjunction can be understood in the sense of elimination, the implication is based on factual meaning of statements etc. Commonly the method of blind attempts is employed, which should not have place in the world of logic (Peregrin[2], 2004).

The second concept considers the logic as a formal science, studying the ways of drawing conclusions from pre-defined premises. Frege and other thinkers, the founders of formal logic, arrived at the conclusion that even the structure of language and our reasoning should be isolated and ‘mathematicised‘, so that they could be analysed through already verified means of mathematics. This brings partial reglementation to the matter (Peters[3], 1998).

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2 The Characteristics of Logical Thinking

Exist more such definitions of logical thinking, and I find it difficult to simply adopt one of them. I have thus decided to specify my own definition of the notion of logical thinking.

2.1 The Author´s Concept of Logical Thinking

On the basis of the individual approaches published I conclude, that it is not possible to exactly define and consequently ´measure´ logical thinking as a whole, but it is necessary to narrow the issue and establish the most significant attributes of logical thinking.

I suggest that *logical thinking is a proces in which the individual looks back from the content of particular statements, and consistently employs particular inferences, so that he arrives at a correct conclusion. The indisputable partial steps of the process create a link between the assumptions and the conclusion through a chain of these inferences.*

With the construction of the test for measuring the level of logical thinking, I focus on the ability of abstraction and correct inference. Given that the pupils face abstraction not only within mathematics, but also geometry, I have incorporated in the test the items that aim at seeking numerical or geometrical regularities. My test (supplement 1), though called simply ´the test of logical thinking´, focuses namely on the ability of abstraction and the ability of correct reasoning.

3 The research issues, aims and hypotheses

1. Selecting on the basis of suitable criteria the games that develop logical thinking with pupils, and specifying the benefits of their implementation to the instruction of mathematics.
2. Finding and describing the factors influencing the effectiveness of logical games´ incorporation in teaching mathematics (namely the individual IQ, the school´s evaluation and the type of school).
3. Drawing up a test assessing the level of logical thinking of pupils.
4. Finding out what factors have impact on pupils´ logical thinking.
5. Finding out whether positive shifts in logical thinking of pupils are monitored after medium-term application of logical games in teaching mathematics.

In these targets were compiled some basic hypotheses that remain divided.

H_1: Amongst the external factors (intelligence, type of school, school evaluation), the predominant part is taken by the intelligence of an individual.

H_2: The level of logical thinking of an individual depends on external factors (intelligence, type of school, school evaluation).

H_3: The capacity of an individual to play mathematical games (Mastermind, NIM, Sudoku) depends on external factors (intelligence, type of school, school evaluation).

H_4: Through medium-term influence on pupils by means of the application of mathematical and logical games, it is possible to reach positive changes in their logical thinking.

3.1 The selection of respondents

The respondents were chosen by multilevel random selection. From the basic category (all primary or secondary pupils over the age of eleven), twenty schools were drawn in the region of Ústí nad Labem, which were then addressed and asked for cooperation. That differed in compliance with the character of research.

	Primary school	Secondary school	Total
Experimental group	60	48	108
Control group	59	47	106
The other pupils	88	127	215
All respondents	207	222	**429**

Tab. 1 Numbers of respondents

3.2 The questionaire research and its statistic processing

The footing for evaluating the level of logical thinking was obtained through printed quetionaires. It was not possible to make use of the existing questionaires or tests of logical thinking, as those do not reflect the level of logical thinking in the way I had adopted for the purposes of this work. A new original test of logical thinking was thus drawn up (see the suplement 1) in two versions; once as a pre-test, then as a post-test.

Each logical test included 12 main items, some of which were further segmented. On the whole, each respondent was solving 22 items, which could be divided into three domains:

- Search for Numeral Regularities (ability of abstraction) – NR
- Search for Geometrical Regularities (ability of abstraction) – GR
- Ability to draw Correct Conclusions – ACC

Each of these domains contains the same number of items. The answers to the questions with the individual items have been evaluated alternatively:

- 0 – the pupil answered incorrectly,
- 1 – the pupil answered correctly.

If the pupil did not respond to a question, an empty sign was used for the coding. Such way of coding allows this interpretation of results: the arithmetic mean of the values measured is the suitable point estimation of the *p* parameter of the alternative distribution, which is the probability that a randomly chosen pupil will answer the related question correctly.

This way allows to evaluate the level of logical thinking of an individual by means of the evaluating vector in the form (NR, GR, ACC). If a pupil has the evaluating vector of (0.73; 0.20; 0.50), then the first component of the vector reports on the fact that the pupil can for example respond correctly to a question from the field of searching for numeral regularities with the probability of 73%. In my research I have made use of this particular type of logical thinking evaluation, as it allows not just the assessment of the progress or decline as such, but also the evaluation of the individual parts in which the change has occured.

4 The Obtained Results

4.1 The impact of external factors on individual´s ability to play the selected games

In this chapter I focus on the ways the observed factors influence the ability of an individual to play the games, when these are incorporated in lessons. Based on the data obtained in the experiment group the following table has been assembled, describing what dependences have been proved between the particular factors and abilities at the significance level of 5 %.

Types of games/individual factors	Type of school	Individual´s intelligence	School evaluation
Individual´s ability to play Sudoku	No	Yes	No
Individual´s ability to play Mastermind	Yes	Yes	No
Individual´s ability to play NIM	Yes	No	No

Tab. 2 Numbers of respondents in course of the experiment

4.2 Progress in the level of pupils´ logical thinking

As it has been stated, I have focused in my work namely on the development of an individual´s logical thinking with the help of mathematical or logical games. For the purposes of the final testing it was necessary to create an end test of logical thinking, that would be analogous to the entry test of logical thinking. The following table provides the numbers of respondents in both the experiment and the control group for primary and secondary level. Only those respondents are included, who have also sat the pre-test and the post-test of logical thinking.

	Primary school	Secondary school
Experiment group	41	31
Control group	65	38

Tab. 3 Numbers of respondents in the course of the experiment

This part of research is related to the hypothesis claiming that through the influence on pupils by means of medium-term application of mathematical and logical games, it is possible to reach positive changes in their logical thinking. The verification of this hypothesis has been carried out sepatately with primary and secondary school pupils.

Primary school

The hypothesis has been proved with the primary school experiment group in all the examined domains. To confirm the fact that the positive changes in the level of logical thinking occured right on the basis of the incorpotration of the selected games in lessons, I carried out analogous testing with a primary school control group. Here, the hypothesis has not been proved in any of the examined domains.

Secondary school

The hypothesis has been proved with the secondary school experiment group in all the examined domains. As it is problematic to make use of a control group (see above), these results can only be considered as approximate. Nevertheless, I state that with the control group the hypothesis has not been proved in any of the examined domains.

It is remarkable that though the games were played with the pupils in only ten lessons, there was a statistically significant improvement observable in all the examined domains (with the exception of searching for numerical regularities with primary school pupils). It is thus possible to confirm the hypothesis H_4, that through medium-term influence by means of the application of mathematical and logical games with pupils, it is possible to reach positive changes in their logical thinking.

5 Conclusion

With the pupils of the experiment group at primary school level I have found, when comparing the pre-test and the post-test, the statistically significant difference in the level of their logical thinking in the domains of searching for numerical regularities and the ability of drawing right conclusions at the level of significance of 5 %, and with the geometrical regularity at the significance level of 1 %. With the pupils of the control group for primary schools, no statistically significant difference has been found in any of the examined domains.

With the pupils of the experiment group at secondary school level, a statistically significant difference has been found in the level of their logical thinking within all the examined domains at the significance level of 1 %. In the group of secondary school pupils where the experiment was not carried out (it is not possible to take it as a standard ‘control group‘ for the reasons mentioned above), I have not found a statistically significant difference in the level of logical thinking between the pre-test and the post-test in any of the examined domains. It can thus be stated that the hypothesis H_5 has been proved. I thus infer that it is possible to positively influence the level of logical thinking (abstraction and reasoning) of an individual.

These conclusions of mine correspond with the research carried out by Lee (1990). He describes abstraction as integral part of individual´s intelligence, and claims that it is possible to influence it positively. Lee further deals with the researh in the ability of human abstraction with regard to one´s age, and he arrives at the conclusion that along with the age it slightly decreases. He also mentions that the relationship between an individual´s intelligence and the ability of one´s abstraction is invariable throughout one´s life. The strong dependence of an individual´s ability of abstraction on one´s intelligence has also been confirmed in this doctoral thesis.

On the basis of my research, I have become convinced that the use of similar methods would also bring its benefit at universities, namely for the students who prepare for the career of a teacher.

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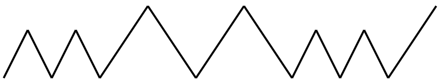
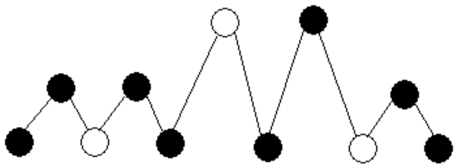
Supplement
The Entry Test of Logical Thinking

Name and surname ……………………………..
Class …………………………………………
School …………………………………………
Mark in Mathematics ……………………….

1) Complete the numeric series by at least three more numbers.
   a) 1, 2, 3, 5, 8, 13, 21……………………………………………………………………
   b) 1, 2, 3, 6, 11, 20……………………………………………………………………
   c) 1, 3, 2, 4, 5, 7, 6, 8……………………………………………………………………

2) Draw the continuation of the following pictures. Add at least three more moves.

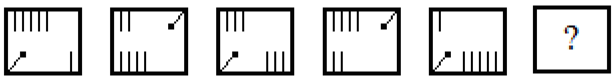


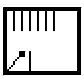
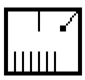
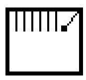
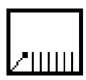


3) The answer to number 52363 is 36325; what is the answer to number 46251? Circle.

A) 25641
B) 26451
C) 12654
D) 51462
E) 15264

4) Which of the pictures completes best the series? Circle.



A)  B)  C)  D) 

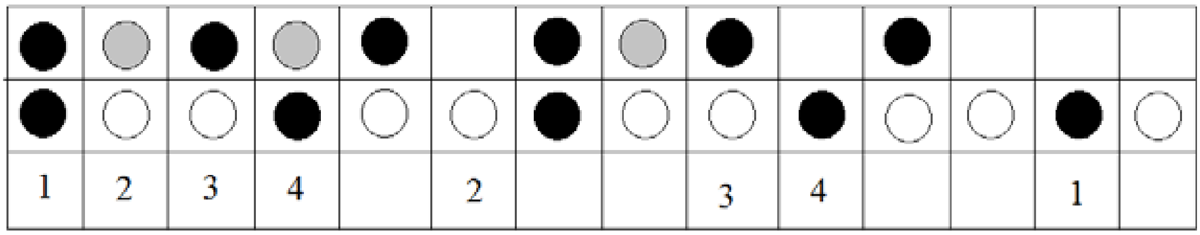
5) Substitute the question marks in the tables by numbers corresponding to the numbers already present.

1	9	**?**	13
2	10	6	14
?	**?**	7	15
4	12	8	**?**

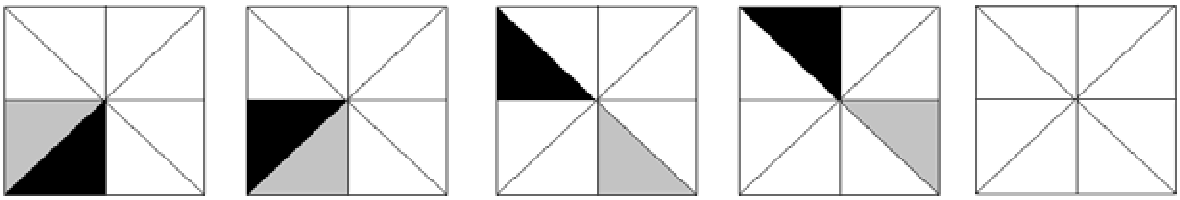
1	4	9	**?**	25	36	**?**

6) Find the regularity and fill in all the empty fields.

a)



b) In the last square, paint in the grey and the black fields.



7) Logical links

a) The natural numbers are 1, 2, 3, 4 etc.

Write down the first five natural numbers divisible by three and four at a time.

…………………………………………………………………………………………...

b) Write down the first five natural numbers divisible by three or four.

……………………………..

c) Is the following statement true? If a number is divisible by six, it is even. ANO - NE

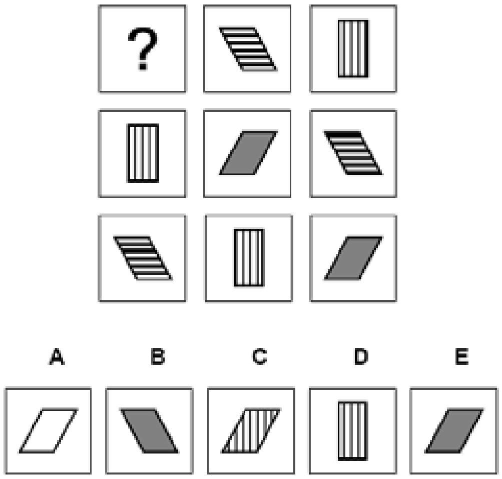
d) Is the statement reversed to the previous one true? ANO - NE

8) Write down the numbers that are even, and fall into the interval between three and sixteen, included.

…………………………………………………………………………………………………

……………………………………………………………………………………….

9) Choose which picture belongs to the field with the question mark (circle).



10) Answer to the following statements. Base your replies on the expressions used in these satements.

a) If you know that each mammal drinks milk and a dolphin is a mammal, what conclusion do you draw from that? ....
……………………………………………………………………………………......

b) Let´s take this statement: Each child has at least one friend. When is this statement not right?

....
....

11) We keep three dogs in the flat, and each dog has its own bed. Alik lies in Bertik´s bed and Rex is not in his own. In which bed is Bertik?

....
....

12) If I do the homework and the training is cancelled, I will go to see my friend. What does the fact mean that I did not go to see my friend? Answer in full sentences. In your reply make use namely of the expressions used in the assignment.

……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………

RESULTS OF THE ENTRANCE EXAMINATION IN RELATION TO SECONDARY SCHOOL-LEAVING EXAMINATION IN MATHEMATICS

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Abstract:
This paper will present the entrance examinations in mathematics for first-year students of selected Bachelor study programmes at the Faculty of Science of the University of Ostrava and it will be compared to the content of the state secondary school-leaving examination in mathematics at its basic difficulty level in years 2011 and 2012. Subsequently, the relationship between the fact of passing the state secondary school-leaving examination in mathematics and results of the entrance examination of students will be monitored.
Classification: Primmary 97A06, Secondary 97B06, 97E06

1 Introduction

As a consequence of implementation of framework educational programmes into the educational system at secondary schools in the Czech Republic, there are significant differences between mathematical competences of secondary schools leavers and, thereby, of applicants for university studies. This is the main reason for gradual inclusion of revision of the secondary school mathematics into the first year of university studies. Year 2011 was the first year of the state secondary school-leaving examination. This made us monitor the relationship between entrance test results and situation whether a student passed the state, or profile, secondary school-leaving examination in mathematics at the same time.

2 Course of Fundamentals of Mathematics

Courses revising selected parts of the secondary school mathematics lessons have always been included into university study plans. Nevertheless, they have often been included in non-mathematical study fields. Concerning the applicants for mathematical and technical majors, a good level of secondary school mathematics knowledge is automatically expected - particularly for the two following reasons:

- majority of applicants for such majors passed the secondary school-leaving examination in mathematics (that was often an obligatory condition within the admission procedure);
- knowledge of secondary school leavers, with respect to unified syllabuses, were comparable.

First-year-students of university studies continued directly with the so called higher mathematics.

Then, the educational (curriculum) reform has entered into such situation. The Czech Republic experienced a gradual transfer from unified syllabuses for all schools to creating and implementing framework educational programmes into school educational programmes and into courses as such.

First framework educational programmes for secondary schools were approved in 2007, and in 2009 first groups of secondary school students started to be learnt in their first years according to adequate school educational programmes[1].

Even though the framework educational programmes for particular fields describe obligatory and recommended extra topics, they provide schools with – contrary to the unified syllabuses – greater flexibility while incorporating these topics into particular school educational programmes. However, this causes even bigger differences in the entrance knowledge and skills of secondary school leavers, notwithstanding, in the same fields of study. As a consequence of the above mentioned, there is also rather varying knowledge in mathematics with the first-year university students (for more see [1] and [2]).

This was the main reason for why the above mentioned courses focusing particularly on revision of the secondary school mathematics have entered even the majors concerning informatics and mathematics at the Faculty of Science, UO. First as compulsory elective courses and gradually, due to individual reaccreditations, it was necessary to rank them among compulsory courses.

2004/2005	Course was ranked among compulsory elective courses for majors concerning geographical and applied mathematics study fields.
2006/2007	Course was added as a compulsory elective course for major in Informatics and for double-major Bachelor degree programmes.
2008/2009	Course was ranked among compulsory courses for the major of Applied Informatics.
2010/2011	Course was ranked among compulsory courses for the major of Investment Consultancy and Mathematics (double-major).
2012/2013	Course was ranked among compulsory courses for majors of Applied Mathematics and Applications of Mathematics in Economics

Ta.b 1 The course Fundamental of Mathematics in the period 2004 – 2013.

In the academic year of 2010/2011 – when the course was ranked among compulsory courses of even the first mathematical study fields – students were allowed to pass the course, and thus to obtain credit, via the entrance test. Main aim was to test the knowledge and abilities of students in selected parts of secondary school mathematics, to identify problematic areas of each student at the beginning in order to be revised and to enable those who managed the curriculum without problems to obtain the credit.

3 A few words on state secondary school-leaving examination

After many years of discussions (serious discussions on changing the concept of the secondary school-leaving examination and introducing the so called „state secondary school-leaving examinations“ started already in the mid-1990s), finally, in 2011, the first state secondary school-leaving examinations were carried out. They were held in two optional levels of difficulty (basic and higher).

[1] Implementation of Framework Educational Programmes into School Educational Programmes of secondary schools is planned in four stages and the last of them is planned in the year of 2012.

State secondary school-leaving examination consists of two parts:
1. Common part = state.
2. Profile part = school.

YEAR	COMMON PART	PROFILE PART
2011	**2 compulsory examinations**: 1. Czech language and literature 2. foreign language or mathematics	**2 – 3 compulsory examinations:** - determined by school head master
	max. 3 non-compulsory examinations: - out of the offer: Czech language and literature, foreign language, mathematics, civics and social sciences, biology, physics, chemistry, history, geography, history of art	**max. 2 non-compulsory examinations** - offer determined by school head master

Tab. 2 The structure of the start-up phase[6].

State secondary school-leaving examination in mathematics can be chosen as a compulsory exam as well as a non-compulsory exam. A student has the option to choose from two levels – basic and higher. School-leaving examination in mathematics is carried out in the form of a didactic test. Almost half of the didactic test at basic level consists of open questions, however, there are many closed questions and it also involves a couple of matching questions.

Area	in %	Area	in %
Numeric study	5-10	**Sequences and financial mathematics**	5-10
Algebraic expressions	10-20	**Planimetry**	10-20
Equations and inequalities	15-25	**Stereometry**	10-20
Functions	10-20	**Combinatorics, probability, statistics**	5-15
Analytic geometry	5-10		

Tab. 3 Areas of mathematic requirements and their approximate representation in the didactic test[5].

Generally, these questions can be grouped into four basic thematic areas, namely into algebra, mathematical analysis, geometry and others.

	2011	2012
Algebra	34.62%	26.92%
Mathematical analysis	19.23%	19.23%
Geometry	26.92%	34.62%
Others	19.23%	19.23%

Tab. 4 Thematic areas and their percentage representation in the didactic test in mathematics at basic level of difficulty.

<table>
<tr><td rowspan="3">Tested course</td><td colspan="5">Limit for being successful at exams as for the common part of secondary school leaving exam</td></tr>
<tr><td rowspan="2">Limit for being successful at exam</td><td colspan="4">Percentage points converted to marks
(upper limit of the interval in percentage points)</td></tr>
<tr><td>4
(Satisfactory)</td><td>3
(Good)</td><td>2
(Very good)</td><td>1
(Excellent)</td></tr>
<tr><td>Mathematics compulsory course</td><td>33%</td><td>51 %</td><td>68 %</td><td>84 %</td><td>100 %</td></tr>
<tr><td>Mathematics non-compulsory course</td><td>33%</td><td colspan="4">Not converted</td></tr>
</table>

Tab 5 Criteria for being successful at the secondary school-leaving examination in mathematic 2011-2012 [5].

In 2011, almost 99 thousand of students registered for the state secondary school-leaving examinations. 19.5 % out of them failed. *The highest failure rate was experienced – as expected – by graduates of secondary vocational schools and of post-secondary studies. The failure with post-secondary students was almost 44 percent; and regarding the apprentices, about one third of them failed. On the other hand, the failure with grammar schools was only 5.1 percent.* [7]

State secondary school-leaving examination in numbers 2011[7]:

98 762 students applied for the state secondary school-leaving examination**.**

9 600 students were not admitted to the exam.

70 722 secondary school students passed the exam.

17 176 students failed the exam.

State secondary school-leaving examination in numbers 2012[4]:

95 191 students applied for the state secondary school-leaving examination.

9 703 students were not admitted to the exam.

69 569 secondary school students passed the exam.

19 816 students failed the exam.

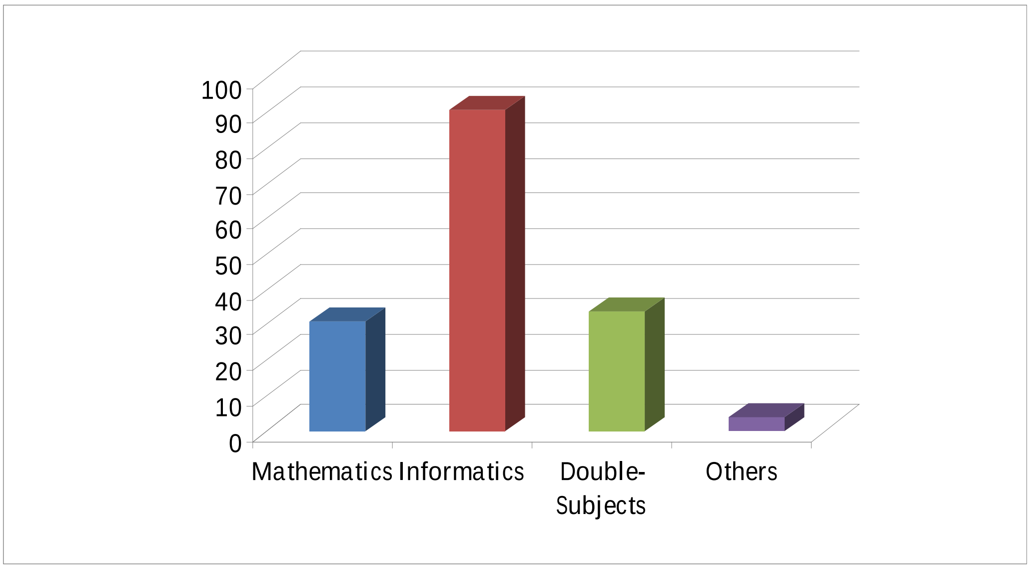
Within the Moravian-Silesian region, there were 5207 students participating/passing the state school-leaving examination in 2011 and there were 5267 students of the same in 2012. Higher level of difficulty was chosen only by 14.04 % of students in 2011. After the experience from the first year of this state exam, the higher level of difficulty was chosen only by 2.81 % of students in 2012.

The success rate in mathematics in the Moravian-Silesian region was 58.35 % in 2011 and 55.31 % in 2012. The entire statistics was based on materials from CERMAT organization as the organization published this data on the aktualne.cz server in 2012.

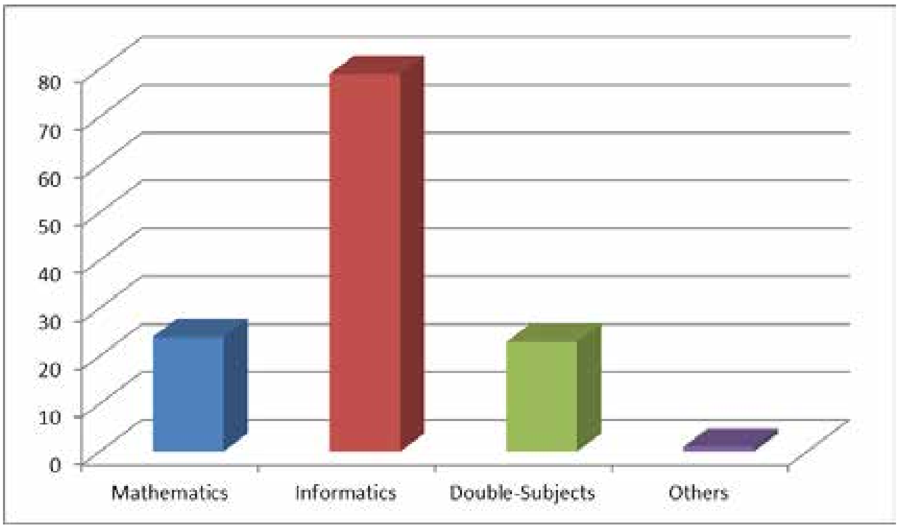
Didactic tests in mathematics at its basic level in 2011 and in 2012 slightly differed, mainly in types of tasks. Table no. x shows that algebraic tasks prevailed in 2011, whereas in 2012 there was a shift to a higher number of geometric tasks. In that year students also showed their highest error rate in geometric part [4]. Students in the Moravian-Silesian region were approximately as successful as they were in the previous year despite the higher number of geometric tasks.

4 Relationship between entrance examination results and secondary school-leaving examination

In spite of the fact that the Fundamentals of Mathematics course was introduced as an obligatory course for all mathematical study fields only in academic year 2012/2013, majority of students involved in mathematical study fields chose this course when selecting compulsory elective courses even in two previous years. The structure of respondents, thus, did not actually change a lot during the respective period of time. Majority of respondents consists of students of informatics fields of study, specifically 56 – 62 % out of the total number. The reason is a much higher interest among applicants for these study fields; it belongs to the most massive ones at the Faculty of Science, University of Ostrava. Double-major Bachelor degree programmes in combination with mathematics and single-major mathematics fields of study are represented similarly in both years, each of them ranging from 18 to 21 %. Other fields of study are represented by a couple of students.

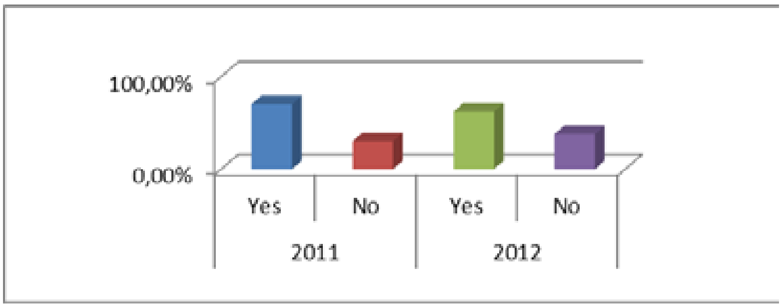


Tab. 6 The structure of respondents according to study fields in the year 2011.



Tab. 7 The structure of respondents according to study fields in the year 2012.

In academic year 2011/2012, 70 % out of all entrance test participants took the secondary school-leaving examination; in academic year 2012/2013, number of such exam participants decreased to 62 %.



Tab 8 Percentage of students took the secondary school-leaving examination.

Structure of the entrance test questions and representation of particular areas, except for one basic task in complex numbers, are identical to the structure of the didactic test in mathematics in basic version.

	Number of tasks	Number of points %
Algebra	7	34%
Mathematical analysis	6	28%
Geometry	6	25%
Others	4	13%

Tab 9 The structure of the didatic test in mathematics in basic version.

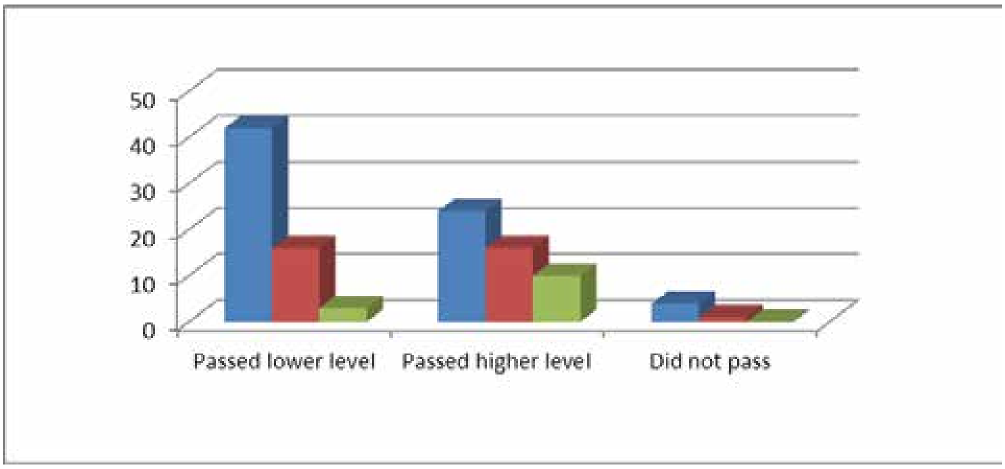
Ration of points is also identical to scoring in the didactic test in mathematics in 2011; however, the area of mathematical analysis (sequence, in particular) is slightly enhanced to the exclusion of other areas (i.e. financial mathematics, combinatorics, probability and statistics). Second difference is in selection of geometric tasks; in contrast to the didactic test of the state secondary school-leaving examination, the entrance test focuses on analytic geometry only.

We monitored the success rate in three categories:
Achieving at least 33 % of points with regard to the limit of the success rate at the school-leaving examination in mathematics at basic level.
Achieving at least 51 % of points with regard to the credit system ECTS.
Achieving at least 75 % of points with regard to the current condition of the possibility to obtain credit at the beginning of a semester via the entrance test.

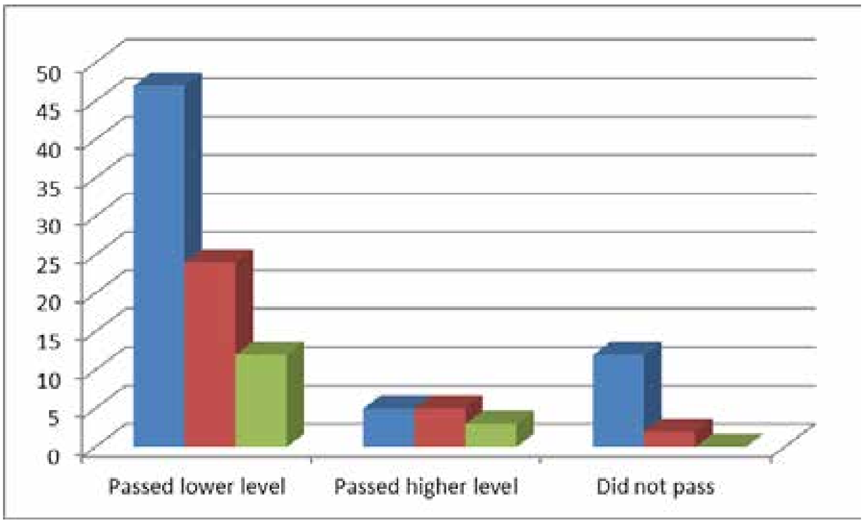
	Success rate regarding the state secondary school leaving exam 2011/2012			**Success rate regarding the state secondary school leaving exam 2012/2013**		
Number of points %	**State secondary school-leaving exam**	**State secondary school-leaving exam – higher level/profile**	**State secondary school-leaving exam not taken in mathematics**	**State secondary school-leaving exam**	**State secondary school-leaving exam – higher level/profile**	**State secondary school-leaving exam not taken in mathematics**
33%	42	24	4	47	5	12
51%	16	16	1	24	5	2
75%	3	10	0	12	3	0

Tab. 10 The success rate regarding the state secondary school leaving exam.

With regard to high degree of similarity between the didactic test of the state secondary school-leaving examination and the entrance test, to higher number of students with school-leaving examination in mathematics among students and also to the fact that students are allowed to use own „cribs“ with listed formulas and relations when writing their entrance test, we expected that majority of students would achieve at least 33 % out of the total. This assumption, however, has not been met. In academic year 2011/2012 this limit was reached by 75 % of students who passed the state secondary school-leaving examination in mathematics at higher level or the profile exam in mathematics and only 49.41 % of students who passed the state secondary school-leaving examination in mathematics at its basic level only. Even worse results were achieved in academic year 2012/2013 - only 37.01 % of students who passed the lower level of the state secondary school-leaving examination in mathematics achieved at least 33 % of points in the entrance test. Comparison with students choosing the higher level, with regard to a large drop in students choosing the higher level of the state secondary school-leaving examination in mathematics, is irrelevant (for more see the Table 11 and the Table12).

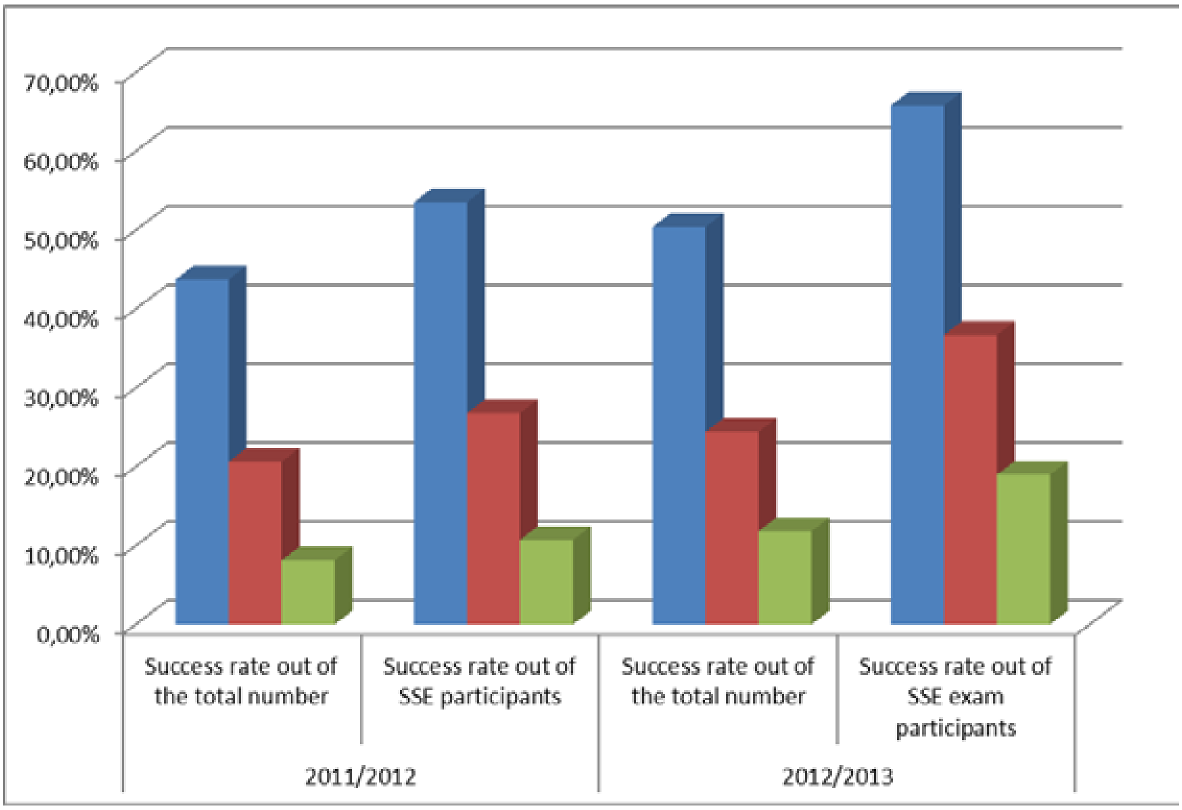


Tab. 11 The success rate regarding the passing exam in the year 2011.



Tab. 12 The success rate regarding the passing exam in the year 2012.

Monitoring the results of all participants of the secondary school-leaving examination regardless the exam level, one can notice that their success rate in achieving 33 % of points in the entrance test in the monitored period of two years ranged from 53.57 % to 65.82 %; speaking about achieving 51 % of points it ranged from 26.79 % to 36.71 % (see the Table 13).

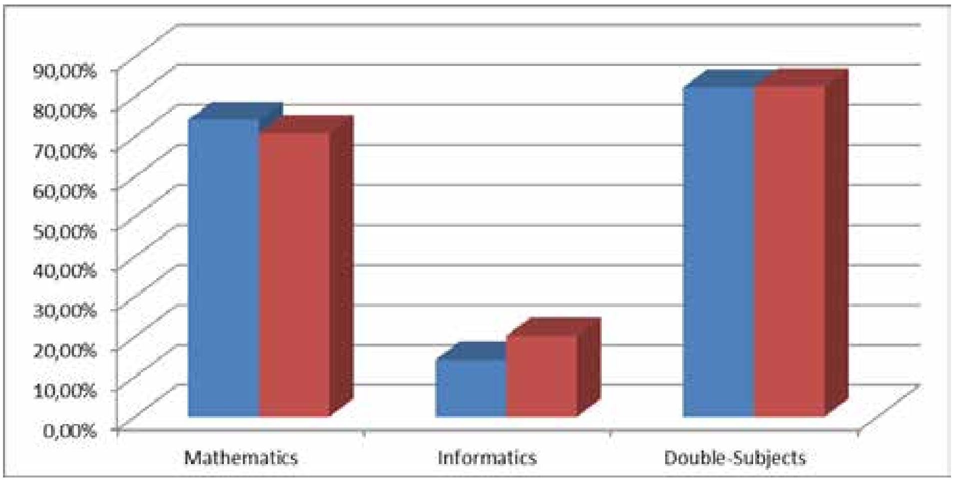


Tab. 13 The success rate out of the total number or SSE (i.e. secondary school-leaving exam) participants in years 2011-2013.

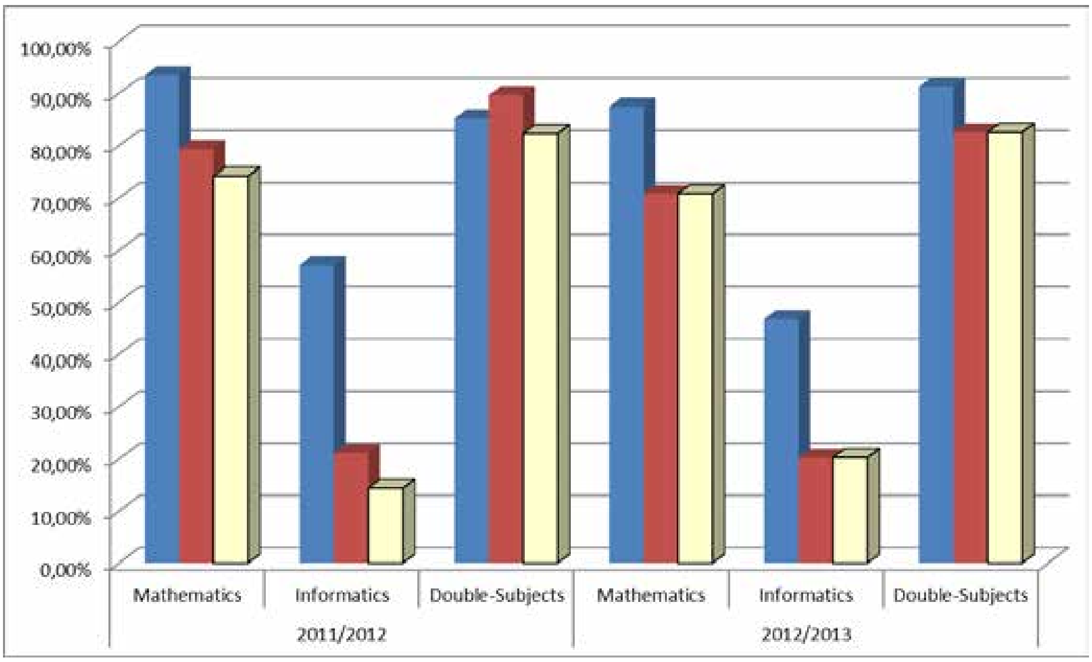
Monitoring the success rate of individual fields of study (i.e. single-major mathematical fields of study, double- major mathematical fields of study and informatics fields of study) we get to surprising results. What is really striking is the difference between students of Informatics and students of two other groups (see the Table 14 and the Table 15). We supposed that the reason for that could be a lower number of school-leaving exam participants focussing on informatics field of study, however, it did not turn out to be right. If we relate results of the entrance test only to students who took the secondary school-leaving examination in mathematics, the difference is still striking (see the Table 16). This despite the fact that students of these fields of study passed, in most of the cases, the state secondary school-leaving examination, it means absolutely identical didactic tests.

	2011/2012			2012/2013		
	Mathematics	Informatics	Double-major programmes	Mathematics	Informatics	Double-major programmes
students in total	31	91	34	24	79	23
33%	23	13	28	17	16	19
51%	13	3	16	12	5	12
75%	4	0	8	8	2	5

Tab 14 The number of successful students at the entrance test in particular point categories by the field of study.



Tab 15 The ratio of successful students – 33 % as the limit.



Tab 16 The success rate regarding the entrance test by study fields.

5 Conclusion

With regard to high degree of similarity of the entrance test with topics and structure of the didactic test in mathematics at basic level, we expected that majority of students who had taken the secondary school leaving exam in mathematics in one of the possible levels given would reach at least the limit of 33 % of points in the entrance test. This assumption, however, has not been proved. Only 53.57 % of this school-leaving exam participants reached this limit in 2011; in 2012 there was a slight improvement to 65.82 %. Another surprising finding is a big difference in the ratio of these „successful“ students in particular groups of study fields. In both years there was a striking difference between the results of students – school-leaving exam participants focussing on informatics and other fields of study. In the monitored period the minimum point limit of 33 % in the entrance test was reached by 75 % of students focussing on mathematical study fields on average and even 86 % of students involved in double-major Bachelor programmes in combination with mathematics. Success rate of students involved in informatics study fields, however, ranged only from 20 to 21 %. But in fact all these students took the secondary school-leaving examination in mathematics and most of them the state secondary school-leaving examination, i.e. absolutely identical didactic tests.

One of the most frequently mentioned reasons for introducing new secondary school-leaving examination was to *obtain such comparable secondary school leaving certificates which could be used by universities as a criterion for admission.* [2] Taking into consideration the above presented results, we regard the option to forgive the entrance exam only on the basis of a successfully passed secondary school-leaving examination in mathematics as impossible. In case that universities would have precise results of secondary school leaving didactic tests at their disposal, its usage for admission procedure at a university could be considered. Nevertheless, it would have to solve the problem of two levels of this secondary school-leaving examination and also to solve the fact that it is carried out in the same period of time as the admission examinations at universities; it means that at present the applicants do not have their results on time and neither do the universities.

Acknowledgements

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LINEAR PROGRAMMING AND HEURISTIC STRATEGIES

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Abstract:
In this article, we analyze the solution of two problems from linear programming from the viewpoint of using heuristic strategies. We wish to illustrate how professional mathematicians apply these strategies in their own work. Finally, we stress the importance of introducing these strategies into school mathematics.
Keywords: Linear programming, heuristic strategy, illustrative figure, auxiliary element, reformulation, experimentation, generalization and concretization, algebraic and geometric method.
MESC: D40, N60

1 Introduction

While many students may consider a mathematics problem to be one of the numbered exercises appearing at the end of a chapter in their text book, to a mathematician, it is something else, characterized by what is present and by what is not. One begins with a question and a goal – usually the answer to the question. What is not present is the path from the question to the answer. Thus, the "solution" to the problem consists not only in an answer to the question, but the discovery or the creation of the path.

There are many strategies available to the problem-solver, such as trial-and-error, guessing-checking-revising, systematic experimentation, reasoning by analogy, generalizing, and concretizing, reformulating the problem, identifying a sub goal, introducing an auxiliary element to name a few. Particularly valuable to the mathematician is the adoption of a geometric or an algebraic plan of attack. In the former, one often begins by drawing a diagram; for the latter, perhaps an equation or a set of equations or inequalities can be written to describe some aspect of the object being studied. In what follows, we will demonstrate some of these strategies using techniques from the subject of linear programming.

Linear programming is a relatively new branch of mathematics. It can trace its roots to George Dantzig (1914 – 2005), an American statistician who was one of a team working for the U.S. Air Force studying efficient allocation of limited resources. In 1947, he rigorously stated the general linear programming problem and developed the simplex method of solution.

A problem often encountered in linear programming is finding the maximum or minimum value of a linear function f of several variables. We can write such a function as

$f = a_1x_1 + a_2x_2 + \cdots + a_nx_n$, where a_i are real numbers.

Almost always the function is accompanied by a number of constraints or conditions that must be satisfied by the variables. For instance, a typical constraint might be that all the x_i must be positive or that $x_1 + x_2 \leq 6$.

There are several methods to attack such problems, including a geometrical approach, an algebraic approach, and the simplex method. We will discuss the first two of these, the last being a special modification of the algebraic method.

2 Methods

1.1 The Geometric Method

Since we will use graphs most easily visualized in a two-dimensional setting, we will deal only with functions of two variables.

Problem 1: Find the point $P(x_1, x_2)$ in the region

$$4x_1 + x_2 \leq 16$$
$$x_1 + x_2 \leq 6$$
$$x_1 + 3x_2 \leq 15$$
$$x_1 \geq 0; x_2 \geq 0$$

For which the quantity $f = 2x_1 + 3x_2$ is as large as possible.

Note that here, the four constraints accompanying the function to be maximized determine and bounded region in the xy-plane. We show the region here:

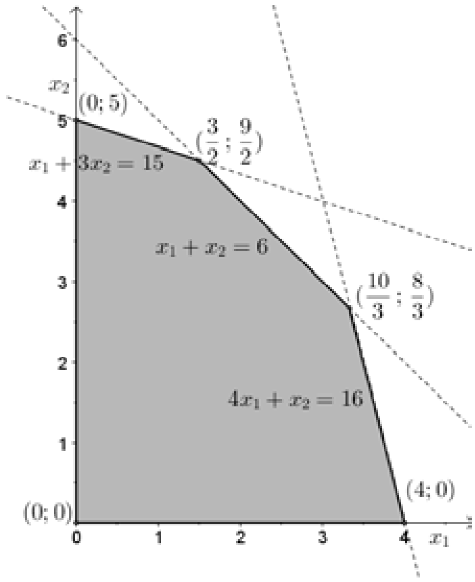

Fig. 1

Solution: If we assign an arbitrary value c to the function f, we obtain the equation of a line with slope $-\frac{2}{3}$, namely $2x_1 + 3x_2 = c$. By using different values of c, we obtain a family of parallel lines.

We show this in the next figure.

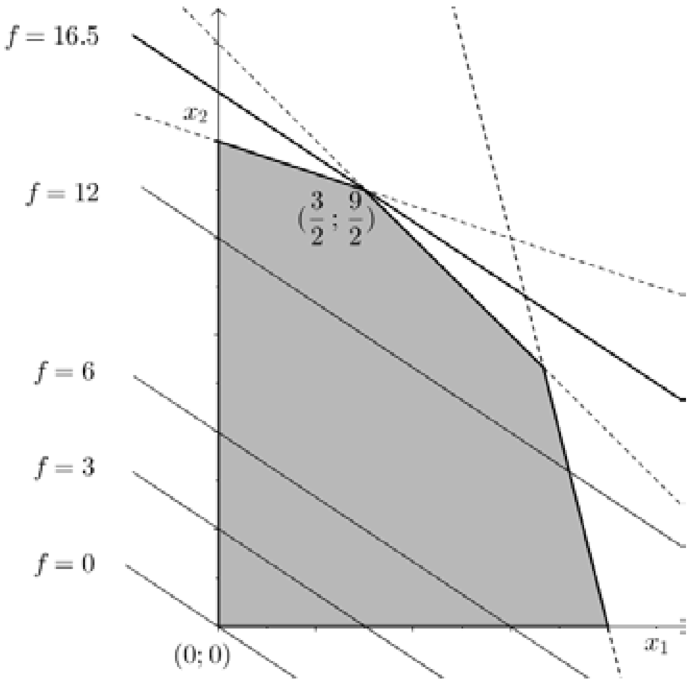

Fig. 2

Lines further away from the origin correspond to larger values of c We see from the figure that the line furthest from the origin touching the region in question touches it at the vertex with coordinates $(\frac{3}{2}; \frac{9}{2})$ Thus, the maximum value of the function f is $2 \cdot \frac{3}{2} + 3 \cdot \frac{9}{2} = 16{,}5$.

We can provide a second solution that makes use of an important theorem in linear programming:

Theorem: Let f be a linear function of two variables x_1 and x_2 defined by

$$f = ax_1 + bx_2$$

If a finite set of linear inequalities in x_1 and x_2 determine a bounded feasible region, then the maximum value of f in that region occurs at one of the vertices and the same is true for the minimum value.

In this problem, we first find the coordinates of the vertices by rewriting the constraint inequalities as equations and then solving them in pairs to find the intersections of the lines. We find that the vertices are located at the points $(0; 5)$, $(\frac{3}{2}; \frac{9}{2})$, $(\frac{10}{3}; \frac{8}{3})$, $(4; 0)$, $(0; 0)$.

The values of f at each of the vertices are

$$f = 2 \cdot 0 + 3 \cdot 5 = 15 \text{ .... at } (0; 5),$$
$$f = 16{,}5 \text{ .... at } (\tfrac{3}{2}; \tfrac{9}{2}),$$
$$f = 14{,}66 \text{ .... at } (\tfrac{10}{3}; \tfrac{8}{3}),$$
$$f = 8 \text{ .... at } (4; 0),$$
$$f = 0 \text{ .... at } (0; 0).$$

Thus, we see that the maximum value of f is 16.5, achieved at $(\frac{3}{2}; \frac{9}{2})$. We also see that the minimum value is 0, taken on at (0; 0).

Remark: If we want to use the geometric method to solve a problem, we must reformulate it from algebraic language to that of geometry. However, this raises a difficulty. The geometric method makes heavy use of diagrams and very few can either imagine or sketch diagrams in more than two dimensions. Thus, we are limited to using the geometric method to situations where there are only two variables. For more than two variables, we must depend on either the algebraic method of the simple method. At first, it seems then that the geometric method is of little use. This is not the case, since insights and methods of attacking a problem developed in the two-variable case can often be adapted to more general situations. We see this in figure 1, which we can call an **Illustrative Figure**. The lines $c = 2x_1 + 3x_2$ which we call level lines are **auxiliary elements**. With the help of these level lines, were able to see that the maximum value of the function occurred at a vertex of the feasible region. The figure guided us to this conclusion because we saw that if a level line did not meet the figure at a vertex, then we could choose a different line with a higher value of c, thus giving us a greater value for f Figure 2 provided the solution to the problem, so we can refer to it as the **Solution Figure.** We point out that since the problem was originally stated in algebraic terms but solved by a geometric method, it is important to return to the original form and state the answer in the “original language” of the problem. Geometric method gave us a valuable insight; we now can apply an algebraic method by finding all the vertices and testing the value of f at each.

2.2 The Algebraic Method

Once more, we begin with a problem.

Problem 2: Maximize the function $f = 6x_1 + x_3$, subject to

$$2x_1 + x_2 + x_3 = 10$$
$$x_1 + 4x_2 \leq 12$$
$$\text{and } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Solution: Our first step is convert any of the constraints are appear as inequalities to equations. We do this by introducing new variables, which we call *slack variables*. Here we introduce the slack variable x_4, which we require to be non-negative and rewrite $x_1 + 4x_2 \leq 12$ as $x_1 + 4x_2 + x_4 = 12$. The slack variable x_4 is an **auxiliary element**. It was not part of the original formulation of the problem and although it will prove valuable, it will not appear as part of the solution.

We have thus **reformulated** the original problem as

Problem 2a: Maximize the function $f = 6 \cdot x_1 + 0 \cdot x_2 + x_3 + 0 \cdot x_4$, subject to

$$\begin{array}{ccccccccc} 2x_1 & + & x_2 & + & x_3 & & & = & 10 \\ x_1 & + & 4x_2 & + & & & x_4 & = & 12 \end{array} \qquad (1)$$
$$\text{and } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

This is a more general problem than Problem 2. That the involved only three variables (and so could be considered geometrically to be in three-dimensional space) while the new problem has four and so is in four-dimensional space.

In system (1), we have a set of two equations in four unknowns, plus the added fact that all the unknown quantities must be positive. We can solve this system by expressing two of the variables in terms of the other two. These latter two are considered to be arbitrary parameters. The solution we obtain is called a *complete solution.* From system (1) we easily obtain

$$x_3 = 10 - 2x_1 - x_2$$
$$x_4 = 12 - x_1 - 4x_2$$

If we set the parameters x_1 and x_2 equal to 0, then we obtain the feasible basic solution $(0;\ 0;\ 10;\ 12)$ which gives a value of 10 for f.

We now seek another complete solution. By performing elementary row operations, we transform the augmented matrix of system (1) as shown here:

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 10 \\ 1 & 4 & 0 & 1 & 12 \end{bmatrix} \longmapsto \begin{bmatrix} \frac{7}{4} & 0 & 1 & -\frac{1}{4} & 7 \\ \frac{1}{4} & 1 & 0 & \frac{1}{4} & 3 \end{bmatrix}$$

This second matrix corresponds to the equations

$$\frac{7}{4} \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 - \frac{1}{4} \cdot x_4 = 7$$
$$\frac{1}{4} \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + \frac{1}{4} \cdot x_4 = 3$$

which we can rewrite as

$$x_2 = 3 - \frac{1}{4}x_1 - \frac{1}{4}x_4$$
$$x_3 = 7 - \frac{7}{4}x_1 + \frac{1}{4}x_4$$

by setting x_1 and x_4 to 0, we obtain the second feasible basic solution $(0;\ 3;\ 7;\ 0)$ which corresponds to $f = 7$.

In the same manner, we can choose x_2 and x_3 as parameters and obtain the equations

$$x_1 = 5 - \frac{1}{2}x_2 - \frac{1}{2}x_3$$
$$x_4 = 7 - \frac{7}{2}x_2 + \frac{1}{2}x_3$$

giving us the third feasible basic solution $(5;\ 0;\ 0;\ 7)$ corresponding to $f = 30$.

The last feasible basic solution is $(4;\ 2;\ 0;\ 0)$ which corresponds to $f = 24$. There are also two other basic solutions, but neither is feasible, for each contains at least one negative entry. Since we can show that the feasible set is bounded, we can now state that the answer to **Problem 2a** is that the feasible basic solution $(5;\ 0;\ 0;\ 7)$ corresponding to $f = 30$ maximizes f Finally, we **return** to **Problem 2**, where only the variables $x_1,\ x_2$ and x_3 were present (recall, we introduced the slack variable x_4) and state that the maximum value of f is 30, occurring at $(5;\ 0;\ 0)$.

Remark: In the solution of Problem 2, we introduced the slack variable x_4 in order to change a system of equation and inequality into one of only equations. This gave us a system much easier to work with. From the viewpoint of heuristics, we introduced an **auxiliary element**. This made it necessary to **reformulate** the original problem. Since the new problem was more general than the first, we thus used the strategy of **generalization**. Since the problem involved an extra variable (an extra dimension in the language of the geometric method) we relied on an algebraic solution. Finally, in returning to the original problem we applied the strategy of **concretizing** or **specializing**. This use of generalization and then specialization is common and the two strategies are often found together.

3 Conclusion

These examples from linear programming show a number of heuristic strategies often employed by mathematicians in solving problems. These strategies are valuable and important tools. One of the most important goals of school mathematics is to make our students better problem solvers. Thus, they too must be able to use these strategies. It is our task not only to demonstrate them, but also to give our students practice in their use. We must present our students with problems that are not only interesting (and if possible, of real-world significance) but also of appropriate difficulty and which make use of their present leel of mathematical knowledge and skill.

3.1 A Last Application

We introduce one last problem, one where there is a need to make the transition from a real-world situation to a mathematical problem. We make use of a table to do so.

Problem: Two factories each manufacture three different grades of paper. The company that owns the factories has contracted to supply 16 tons of low-grade paper, 5 tons of medium-grade, and 20 tons of high-grade. It costs \$ 1 000 per day to operate Factory 1 and \$ 2 000 per day to operate Factory 2 Factory 1 can produce 8 tons of low-grade, 1 ton of medium-grade, and 2 tons of high-grade paper each day. Factory 2 produces 2 tons of low-grade, 1 ton of medium-grade, and 7 tons of high-grade paper each day. For how many days should each factory operate to fill the order most economically?

Solution: We present the information in a table:

	Factory 1	**Factory 2**	**Tons Needed**
Low-grade	8 tons per day	2 tons per day	16
Medium-grade	1 ton per day	1 ton per day	5
High-grade	2 tons per day	7 tons per day	20
Daily cost	\$ 1 000	\$ 2 000	

Tab. 1

Let x_1 be the number of days that Factory 1 operates and x_2 be the numbers of days that Factory 2 operates to fill the order.

The contraints are:

$8\,x_1 + 2\,x_2 \geq 16$ (at least 16 tons of low-grade paper are required)

$x_1 + x_2 \geq 5$ (at least 5 tons of medium-grade paper are required)

$2\,x_1 + 7\,x_2 \geq 20$ (at least 20 tons of high-grade paper are required).

$x_1 \geq 0; x_2 \geq 0$ (the number of days of operation must be nonnegative).

We wish to minimize, subject to the above constraints,

$$f = 1\,000\,x_1 + 2\,000\,x_2$$

which is the total cost of operating the two factories.

We use the geometric method. The shaded region in Figure 3 shows the feasible region:

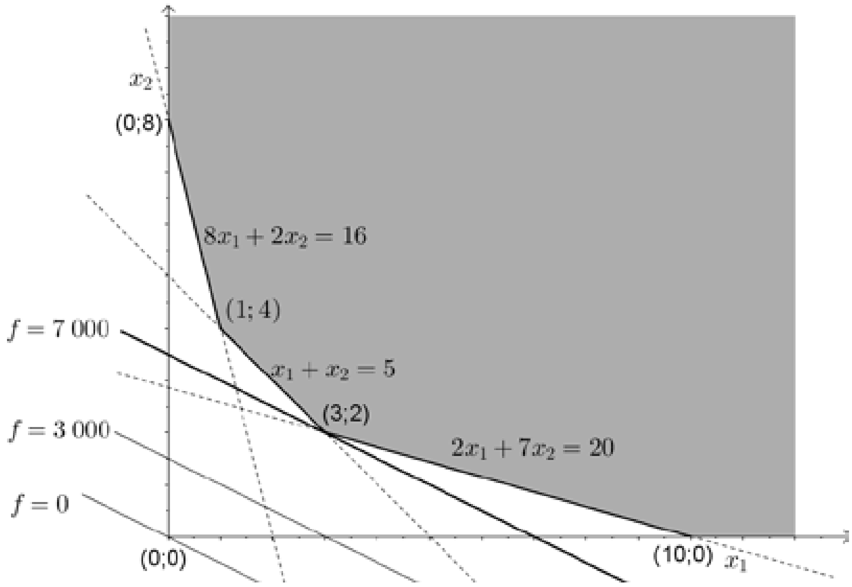

Fig. 3

Since the feasible region is not bounded, we must use level lines. We know that if a minimum exists, it must occur at a vertex. Using c as a parameter, we see that $c = 1\,000\, x_1 + 2\,000\, x_2$ is the equation of a family of parallel lines (also shown in Figure 3). From the figure, it is clear that the line corresponding to $c = 7\,000$ is the "lowest" line with a point in common with the shaded region. Thus, the point $(3; 2)$ minimizes f at a value of $3 \cdot (1\,000) + 2 \cdot (2\,000) = 7\,000$

Answer: To fill the order most economically, Factory 1 should operate for 3 days and Factory 2 for 2 days. The total cost of operation will be $ 7 000.

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USAGE POSSIBILITIES OF E-TESTS IN A DIGITAL MATHEMATICAL ENVIRONMENT

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Abstract:
Didactic tests as we know them serve for determining the quality and the quantity of the students' knowledge. The creation and evaluation of didactic tests are in the focus of special theories. Digital technology has brought and created electronic aids (hardware and software) for testing students. These aids have further possibilities of usage in the education process such as motivation, repetition, exercising, evaluation. They can also be used in new teaching methods such as controlled discovering. The goal of this paper is to show various possibilities of using the free software HotPotatoes and GeoGebra in didactic situations in teaching mathematics on high schools.
Keywords: e-test, new teaching methods
MESC: U70, N80

1 Introduction

In the past decade, digital technologies have become a part of our everyday lives. This environment is new for us, but is natural for the students and it has also appeared in education. A computer with an internet connection can be found in every school, although an interactive whiteboard is nothing new either. It is proper then to make the teachers and didactic researchers engaged in the application of modern digital technologies. That is why new (or updated, “digitalized”) teaching methods and forms appeared in secondary mathematics education (the constructive education method, problem and project based learning, workshop and peer instruction methods). (Lukáč, 2010)

The ICT is present in every phase of a mathematics class: during motivation, exposition, fixation, diagnosis. For determining the quality and quantity of the students’ knowledge, we use traditional tests. With the digitalization of education various electronic tools (hardware and software) aid the measurements. These new digital technologies are excellent for application in the educational process (e.g. during motivation, repetition, exercise or rating).

2 Mathematics education in a digital environment

We can use many digital tools in secondary mathematics education. As hardware equipment a computer and a projector can be used, as well as interactive whiteboards and clickers (equipment used to give answers during e-tests), the document camera, even mobile phones and tablets are applicable. The use of the interactive whiteboard in mathematics education is described by Part Edit and Katarína Žilková. In a classroom equipped with computers, students can learn with their own pace and rhythm, experiment with various applications and come up with theories which can they themselves verify.

We will mention only a few freely available software products used in mathematics education: the plotters Graph (19) and Graphmatica (20), the dynamic geometry software GEONEXT (21), C.A.R. (22), and Euklides (23). The CAS software products are mostly paid, but for example WX MAXIMA (24) and Casyopée (25) are free. We can mention the MS Mathematics software (26), because it is free for Windows operating system owners and its performance can be compared to a graphical calculator.

We can specifically promote the GeoGebra software (27), which is open source and proved to be a very useful tool in creating graphs, interactive mathematical applets and in dynamic geometry applications. Another software, as well freely available and open source, is HotPotatoes (28), which was designed for electronic tests. We outline the use of these two programs later on.

3 The e-tests

Using conventional tests, we can determine the students’ knowledge from a qualitative and a quantitative point of view. The creation and rating of these is in the field of theory of testing. With the digitalization of education various electronic tools (hardware - e.g. clickers, software - e.g. HotPotatoes) aid the measurements. We must be careful what do we use the HotPotatoes or the clickers e-tests for.

According to their use we classify the e-tests as follows: 1. If we are determining the students’ knowledge, then

- we must follow the traditional testing requirements (validity, reliability)
- they contain tasks that are computable in a short amount of time

2. If we want to increase the motivation of the students, then

- using the questions we gather the students’ opinions
- the tests are problem solving oriented
- can be used in controlled discovery

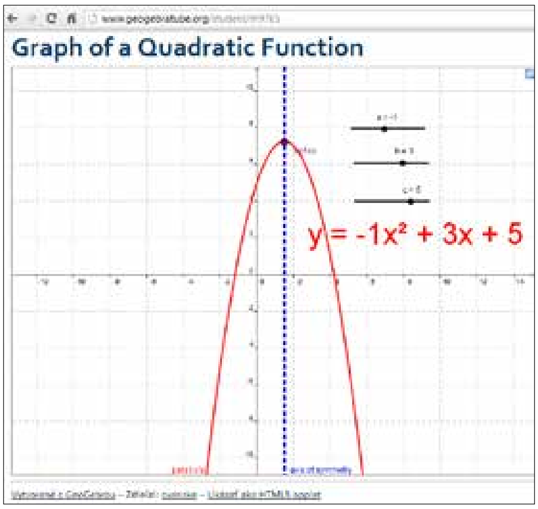
3. If we use it as an interactive worksheet (this is no longer a traditional test, the electronic test ist just a tool here)

- can be used in controlled discovery
- interactive (after incorrect answers, it guides the student with adapted tasks)
- the participants can experiment using GeoGebra or other software

We can widely apply these new digital tools in teaching as well (e.g. motivating the students, repetition, exercise, rating). The clickers used with interactive whiteboards allowed two new educational methods - the peer instruction and the workshop method - to be applicable. These belong to the constructivist methods.

Example: The e-test, as an interactive worksheet

Drawing the graph of a quadratic function and finding its roots often presents a problem for the students. This e-test was created using the HotPotatoes software, but on its first page the GeoGebra software has sketched the $y=ax^2+bx+c$ function based on the a, b, c parameters which the student has typed in.



Pic. 1 GeoGebratoube (13)

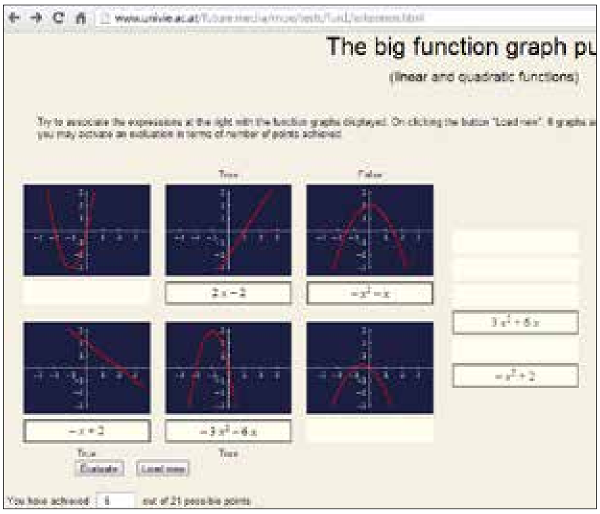
Test question examples:

- What is the value of the c coefficient if a=1, b=4 and the graph does not intersect the x axis?
- What is the value of the b coefficient, if a=1 and the graph intersects the x axis in 0 and 4?
- How does the sign of the a coefficient affect the graph?
- How does the graph change if we change only the c coefficient?

The questions are ordered by difficulty and the students make important discoveries using GeoGebra on how do the a, b, c coefficients affect the graph.

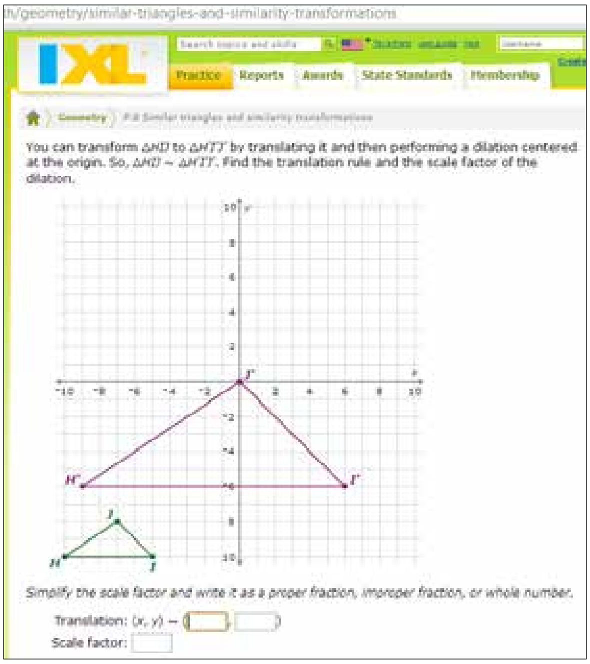
Example: The e-test as a playful knowledge test

There are plenty of websites with tests that help the students to test their collected knowledge (therefore the mark is not the goal).



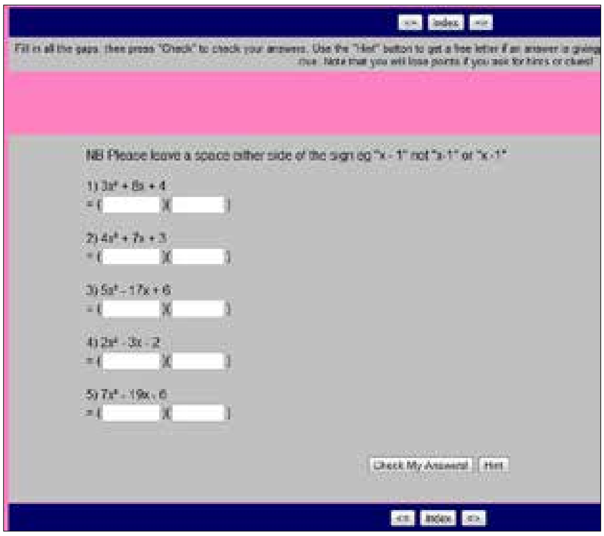
Pic. 2Maths online (14)

On this website of the Vienna University, many collections of Java applets can be found tests which can serve for self-rating of the students by either group work, or using an interactive whiteboard or even individually, if the students have tablets or computers. They are well suited even as preparation at home. This specific test serves for developing knowledge about a linear and quadratic function. The student's task is to assign the correct formula to the given graph of a function.



Pic. 3:IXL Learning (15)

On IXL Learning's website, many tests can be found, which are ordered by thematic groups, from First Grade to Eighth Grade and are further divided into educational topics.

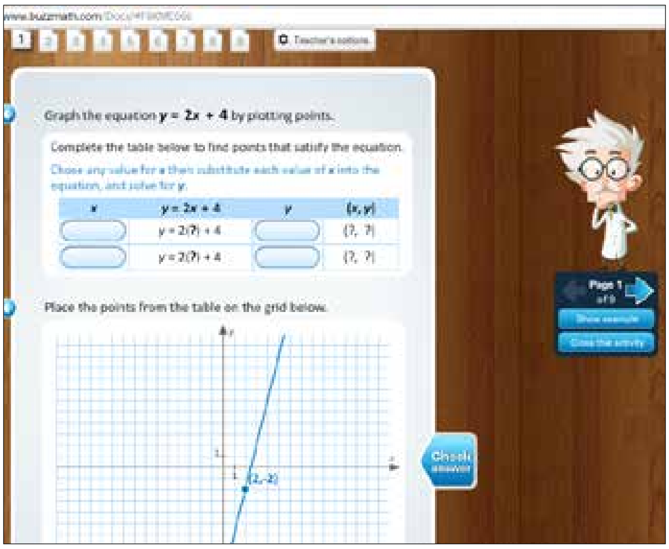


Pic. 4: Factorising Quadratics, HotPotatoes test (16)

Many e-tests can be found on the Internet which were created by teachers using HotPotatoes. Most of these tests have not been reviewed for validity, therefore we must be cautious because some do not fulfill the requirements of a good e-test.

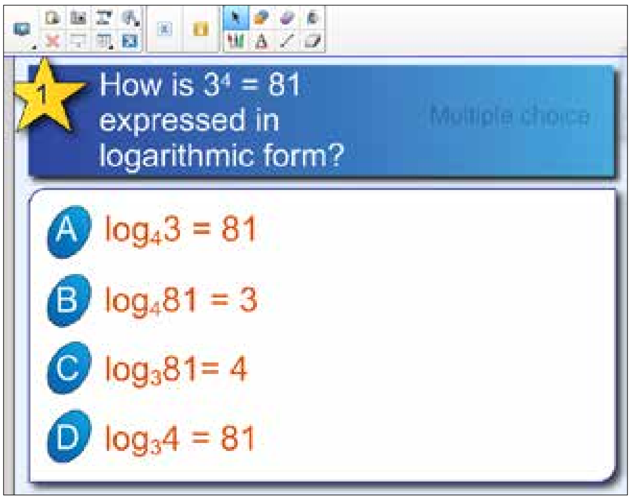
A didactic test guarantees its advantages only if its correctly assembled, properly used and correctly rated. Anyone who wants to create a didactic test must know the correct way how to create it and must be able to measure its properties and be able to modify it so it can be practical and will serve correct information. (Lapitka, 1990)

These rules apply for e-tests as well. During the creation of an e-test, we can not forget what kind of an e-test are we making. One possibility is a classic school test for summative rating (e.g. half-year or a yearly test), which we will only electronically administrate. The other possibility is an e-test for motivation and self-rating of the students during class. In this case, questions must be short, calculations and logical operations must take a short amount of time for the given task. Another possibility is computerized adaptive testing, which we can use for formative rating of the students. This kind of test gives the student his next task based on the answer from the previous task, thus calibrating the the level of his knowledge in the given theme. These kind of tests are very sophisticated and must be created by a team of experts which must have proper software.



Pic. 5: buzzMath (16)

The BuzzMath portal offers a great scale of interactive test for students on elementary schools. They are proper for exercising because it offers a sample task in the beginning and the correct solution if a wrong one was provided. Motivation is increased also by rewarding: the students can play a short game after a certain amount of correct answers.



Pic. 6: SMART Exchange (18)

Another kind of e-tests are tests, which are used on an interactive whiteboard with voting devices (clickers). These e-tests can be used at the beginning of a class for repetition or at the end of a class, whether the students understood everything.

4 Conclusion

At the beginning, experts tried to objectively measure the performance of students and slowly the theory of didactic tests was developed. Mathematics also long uses standard tests for rating students on finals tests or at the end of the 9th grade. Teachers use non-standard half-year or yearly tests.

In the past years information scientists have created a large scale of software products for easier administration of classic school tests. Later on new hardware equipment was added, which could have been used for electronic testing. It was however proved that these tools have much greater usage possibilities than just tests administration. They can be used as an effective aid for innovative teaching methods.

Therefore we can today define the term “e-test” dually:

1. In a narrower meaning, the e-test is an electronically controlled didactic test with an option to enrich it with multimedia elements.

2. In a wider meaning, the e-test is an electronic interactive material based on a system of questions and searching for answers created not only for measuring, but also for reaching educational goals (hence can serve as tools for innovative teaching methods)

Using e-test we are able not just to determine the students’ knowledge, but with these new digital tools we can increase the students’ motivation, use them during repetition, exercise, controlled discovery methods. The e-test is very attractive from the students’ point of view, because the digital world is very close to them.

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WX MAXIMA, http://andrejv.github.com/wxmaxima/
Casyopée, http://www.casyopee.eu/
MS Mathematics, http://www.microsoft.com/education/en-us/teachers/guides/Pages/Mathematics-guide.aspx
GeoGebra, www.geogebra.org
HotPotatoes, http://hotpot.uvic.ca/

LEGO MINDSTORMS ACTIVE SUPPORTS THE TEACHING OF PROGRAMMING

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Abstract:
The paper deals with the use of education building set Lego Mindstorm in gradual education or in the carrier education of teachers at primary and secondary school. There is described the learning possibility which implements theoretical knowledge of algorithm, simple programming language NXT-G and subsequent application of knowledge and skills into higher programming language Bricx.

Key words: Lego Mindstorms, NXT-G, Bricx, teaching of programming
Classification: P40, Q60

1 Introduction

The basic task of teaching programming is to harmonize the content of the curriculum by age students, as well as to achieve a basic level of knowledge of a programming language. We can respect the evolutional stage of pupils, to guide their long-term continual education as well as to adjust the program complexity to their mental abilities. The Lego mindstorms has proved as very outstanding for its HW and SW items. This educational set and programming environment are essentially aimed to the pupils' age of eight. Set consists of programmable microcontroller which is used for simple operated applications. These applications do not depend on computational power of microcontroller but there is significant low price and repeated.

Fig. 1 Part of NXT Mindstorm

There are used embedded devices which are part of everyday life in consumer and industrial electronics. There are usually some specialized microcontrollers, for example for control of engines section and sensors, for control of input and output on display and for control of device´s diagnostic.

2 The education set Lego Mindstorms

Microcontroller's programming has some particularities following from structure of their circuits, from connection of different kinds of external sensors and actuators. The programming language and the simulator have their own particularities. The education set Lego Mindstorm (Fig. 1) presents important asset in the field of teaching of microcontroller´s programming. The set consists of hardware with programmable microcontroller´s part, essential sensors and actuators which are able to scan basic physical quantities of surrounding environment. We can create many interactive applications on the basis of this software. There is possibility to use some programming languages which are adjusted to age and abilities the users. The users of different age and level of programming knowledge are able to create and program behaviour of different vehicles, robots and interactive toys.

The set can be used for IT teaching at primary schools for beginners at the age of 9 to 12. The most suitable programming environment for that age group is NXT-G. It is iconic kind, orders, cycles etc. are assigned by figure 2.

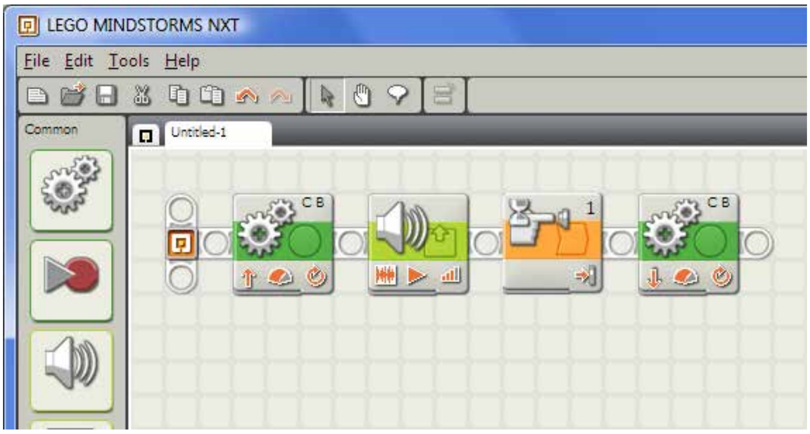


Fig. 1 Develope environment NXT-G

Parts of this programming environment are interactive clues. They consist of manuals for creation of basic robotic chassis. The other parts are pictures of essential programming sequences for input and output devices, for instance sensors, LCD display, direct engines and wireless communication. The advantage of this programming teaching is exchange of flow chart to icons from environment NXT-G. The user is not confronted with errors which occur during error orders. Its attention is aimed at logical order series of flow chart and program.

3 The environment Bricx

The programming language Bricx and set Lego Mindstorms allow the students of secondary schools to achieve practical experience in the field of microcontroller´s programming in the language C++. This software (Bricx) uses the same kind of syntactic order marking and variable quantities as language C++. Moreover, it contains the orders for programming of sensors, actuators, LCD display and wireless communication as well as external periphery (Fig. 3). The set Lego Mindstorms with software Bricx allow us to continue the user´s education in connection with basic iconic programming towards higher programming language. The environment is free and in large measure identical with programming language C++. There is no support of ICT teaching in the environment Bricx from producer of LEGO. There is suitable to create and prepare masterpiece by support. This set is appropriate educational tool not only for software engineer but also ICT teachers. The advantages of using are visual connections of marks which are used for algorithm with icons of program NXT-G. We can use the orders from language Bricx later.

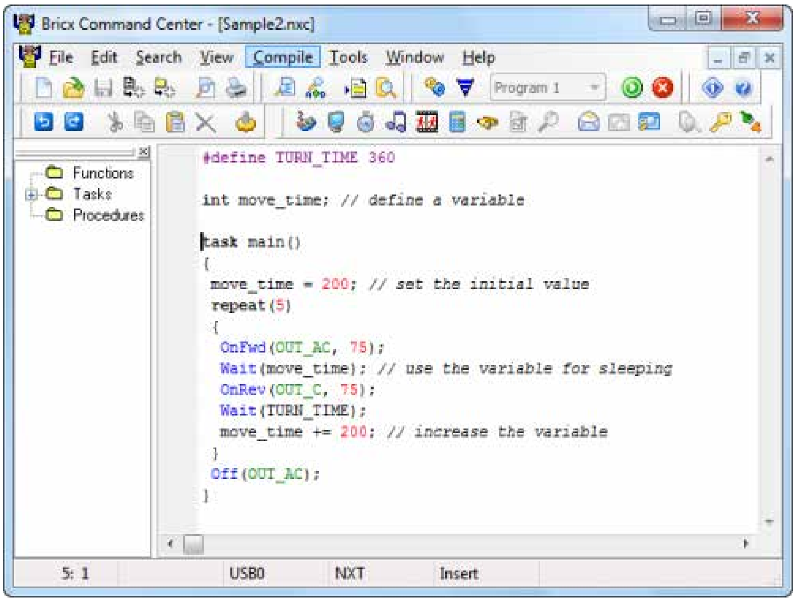
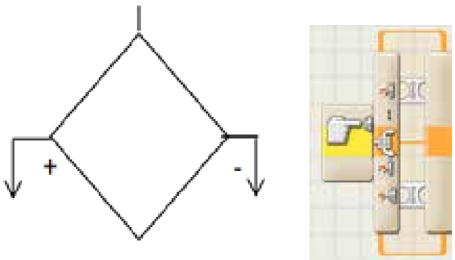


Fig. 2 Develope environment Bricx

The student obtains visual graphic information about relation between theoretical notation of solved problem (icon of algorithm) and icons of orders which are essential to use for program creation. There is made relation which is later matched to written order (Fig. 4).




```
if (Random() >= 0)

{
OnRev(OUT_C, 75);
}
else
{
OnRev(OUT_A, 75);
}
```

Fig. 3 Program branch in icon NXT-G and in syntax Bricx

4 Conclusion

The set Lego Mindstorms presents suitable education tool which supports continual learning in the field of programming from beginning to higher programming language Bricx. The syntax of this programming language and language C++ is large. We can declare that Bricx is its derivate and it alleviates transition to language C++. We obtain supporting tool of teaching programming from order description and explanation or from masterpiece.

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Figure 2.: http://www.nxtprograms.com/help/learn.html

DEVELOPMENT OF PUPILS' DIGITAL COMPETENCES

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Abstract:
The paper deals with the development of pupils' digital competences through an online contest on informatics and computer fluency "iBobor". We focus on the first category of this contest called "Bobrík" which is designed for 3rd and 4th grade primary school pupils. The paper analyses the presence of competences in the area of "Mathematics and work with information" according to the State Educational Programme for the First Stage of Basic Education in the Slovak Republic (ISCED 1) through a number of selected competition tasks.
Key words: digital competence, primary education, informatics contest
MESC: D30, D40

1 Introduction

According to the ISCED 1 (Slovak State Educational Programme), the area of *Mathematics and work with information* involves school subjects *Mathematics* and *Informatics education*. The digital competence includes the ability to understand the information from various ICT sources and use it in various formats.

"The objective of the school subject Mathematics at the first stage of primary school is to develop those abilities of pupils that help them to prepare for individual extracting and applying knowledge."[1] As for the work with information, the mathematics curriculum is oriented to work with tables, graphs and diagrams. Pupils, depending on their capabilities, solve tasks that apply mathematics in real life. They use ICT tools (calculators, computers) to search, process and store information.

"The objective of Informatics education at the first stage of primary school is to familiarize pupils with computers and possibilities of their use in everyday life."[2] Pupils acquire basic computer skills through the use of various applications made appropriate to the age of pupils in terms of their content and user control. As for the cross-curricular relationships, pupils practise basic curriculum from other school subjects (mathematics, native and foreign languages, science, etc.) through various computer applications. They can develop their creativity and an aesthetic feeling through the use of various graphic editors. The Informatics education is delivered in one lesson per week in 2nd, 3rd and 4th grades.

The topic area **Procedures, problem solving, algorithmic thinking** gives pupils the opportunity "to become familiar with specific problem solving procedures through ICT".[3] The

[1] http://www.statpedu.sk/files/documents/svp/1stzs/isced1/vzdelavacie_oblasti/informaticka_vychova_isced1.pdf, p. 13
[2] http://www.statpedu.sk/files/documents/svp/1stzs/isced1/isced1_spu_uprava.pdf, p. 14
[3] http://www.statpedu.sk/files/documents/svp/1stzs/isced1/vzdelavacie_oblasti/informaticka_vychova_isced1.pdf, p.4

biggest benefit of this topic is that pupils acquire the basics of algorithmic thinking and an ability to use ICT in finding solutions to problems. They learn different ways and mechanisms to solve problems from various areas, and also to think about different effectiveness parameters in problem solving.

Pupils obtain similar competences during mathematics lessons and through the topic area **Sequences, relations, functions, tables and charts**.[4] Pupils learn to create simple sequences of objects, recognize and discover a rule of succession. Then they continue alone in creating new elements of such sequences. They learn to sort table data after identifying links among them. Understanding, analyzing and modelling solutions of tasks and problems help pupils to cultivate their skills and creativity. The iBobor contest is one of the ways how get student involved in order to promote the aforementioned competences with use of ICT.

2 The iBobor contest

Informatics contests have an important role in developing pupils' skills to use ICT and solve problems by using a computer. Pupils of primary and secondary schools can join the iBobor[5] (i.e. Informatics beaver contest) in Slovakia. This contest is organized by the Department of Informatics Education at the Faculty of Mathematics, Physics and Informatics of Comenius University in Bratislava in cooperation with an international team of experts. Main principles of the “Beaver” contest structure have been borrowed from the international mathematical contest “Kangaroo”.[6] Pupils of 3rd and 4th grades (the *Bobrík*[7] category) got involved in the competition for the second time in 2012.

The competition consists of four groups of tasks: digital literacy, programming, problem solving and work with data.[8] Types of competition tasks for Bobrík category are tailored to the pupils attending 1st stage of primary school and hence interactive tasks reflect pupils’ playfulness. Pupils can try several solution alternatives and experiment with data in these category tasks. Graphic design of competition tasks is also important - tasks contain colour images that are appropriate to the pupils' age.[9]

According to the rules the Bobrík category has 12 tasks and children have 30 minutes to solve them. The tasks are scored according to their difficulty: the 4 easiest tasks with 3 points, another 4 tasks with 6 points and 4 most difficult tasks are scored with 9 points. In case of a wrong solution, 1, 2 or 3 points get subtracted (-1 for easy, -2 for moderate and -3 for a difficult task). With a starting bonus of 24 points, a pupil can earn 96 points maximum. Easy tasks are called: 1-*Animation*, 2-*Creatures*, 3-*Song* and 4-*Coding*. Moderate difficult tasks are: 5-*Domino*, 6-*Coloured Way, 7-Labyrinth, 8-Robots*. The most difficult tasks are: 9-*Candies*, *10-Where they live*, *11-Repaint*, *12-Little Turrets*. Slovak tasks can be searched and viewed in a tasks archive on the contest’s website (http://ibobor.sk/sutaz_demo/). The Czech version of this contest can be found on

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[4] http://www.statpedu.sk/files/documents/svp/1stzs/isced1/vzdelavacie_oblasti/matematika_isced1.pdf
[5] Bobor in Slovak means beaver. The contest belongs to the international Bebras Contest - http://bebras.org
[6] CARTELLI, A. et al.: Bebras Contest and Digital Competence Assessment: Analysis of Frameworks. In: 24 International Journal of Digital Literacy and Digital Competence, 1(1), 24-39, 2010. 2010
[7] Bobrík in Slovak means little beaver.
[8] KALAŠ, I., TOMCSÁNYIOVÁ, M.: Students' Attitude to Programming in Modern Informatics. In: Proc. of 9th WCCE IFIP World Conference on Computers in Education, Bento Goncalves, Brazil, 2009.
[9] TOMCSÁNYIOVÁ, M: Základy programovania na 1. stupni ZŠ. In: Didinfo 2011. Banská Bystrica: UMB Banská Bystrica, 2011, p. 232-239.2011.

ibobr.cz. The Czechs also included a category for 1st stage of primary schools (*Mini*) in the school year 2012/2013.

In this paper we analyze tasks' solutions of a group of 13 children from St. Vincent Elementary School in Ružomberok. These pupils were intentionally selected based on their computer skills. They obtained from 23 to 88 point in the contest (Fig. 1). The contest tasks show the level of digital and mathematical competencies in several areas. Chart success of individual tasks is shown in Figure 2.

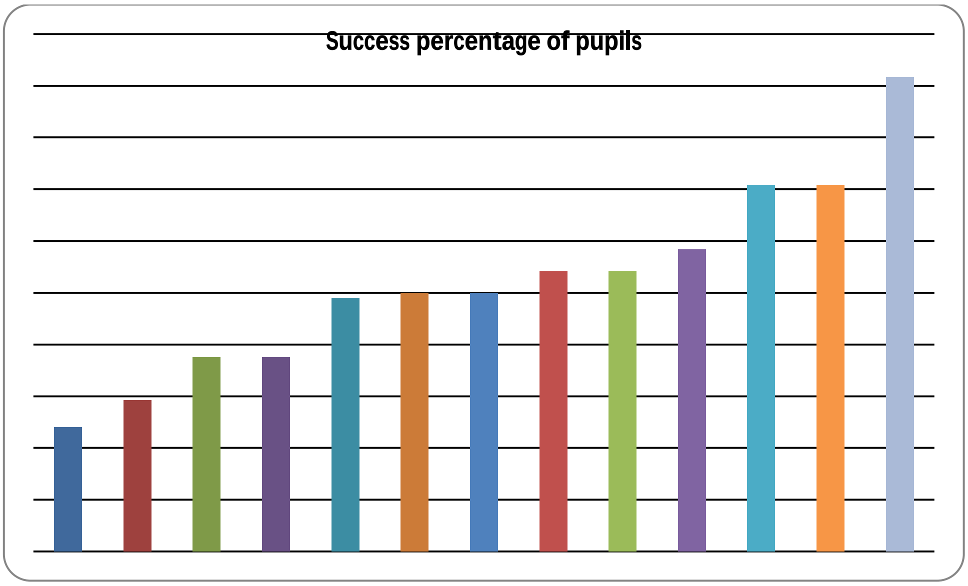

Fig. 1 Success of pupils in contest

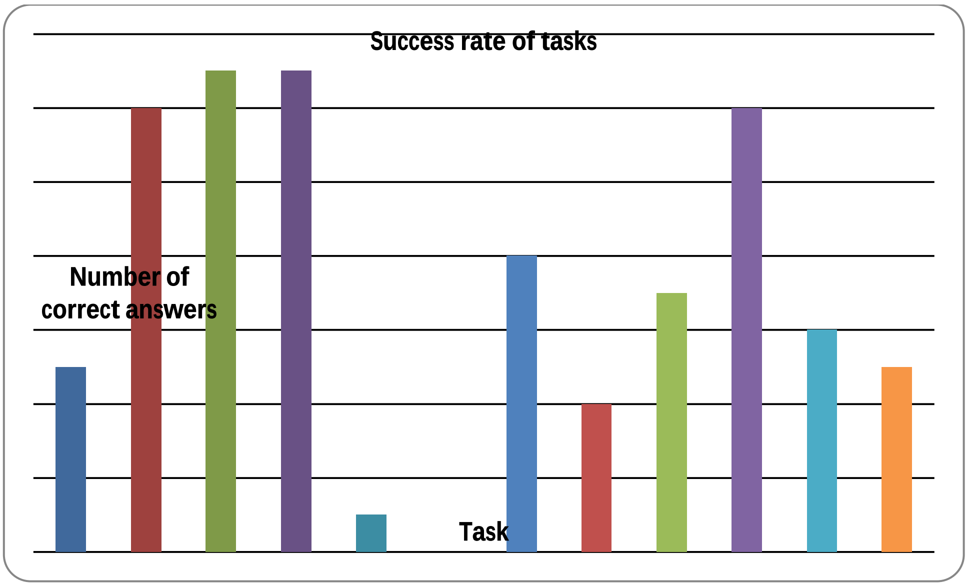

Fig. 2 Success of tasks in contest

In the maths topic area called *Logic, reasoning, proofs*, pupils solve tasks in which they assess veracity of statements from the area of maths or real life situations - whether they are true or false. The standard defines and requires the ability to distinguish simple and reasonable true or false statements.[10]

[10] http://www.statpedu.sk/files/documents/svp/1stzs/isced1/vzdelavacie_oblasti/matematika_isced1.pdf

In the task of moderate difficulty called *The Robots*, (Fig. 3, source: http://ibobor.sk/-sutaz_demo/) children decide whether a statement for each of robots is true or false. For instance, they have to answer how many robots came to the blackboard after these two statements: "If you have green wheels, stop listening!" and "If you have a shoulder, go to the blackboard!" This kind of decision making occurs in children's everyday life. In spite of that, only 4 out of 13 children answered the question correctly.

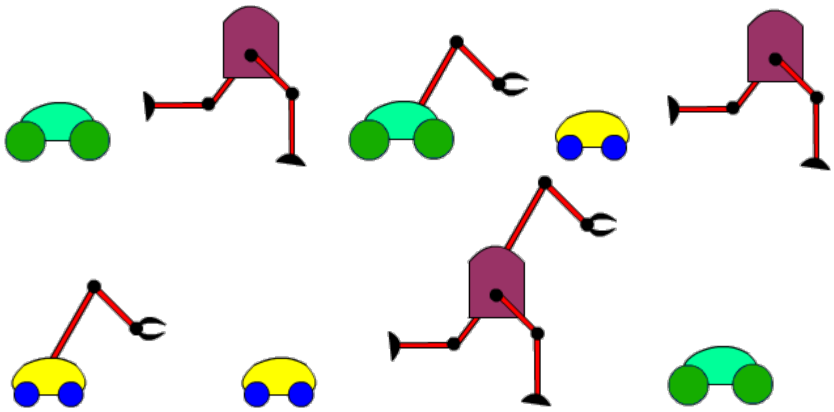

Figure 3 The task Robots

Another task of moderate difficulty called *Coloured way* (Fig. 4) wasn't solved correctly at all. It is because no child ticked two correct options. At the national scale, this task had only 10 % average success in Slovakia (source: ibobor.sk).

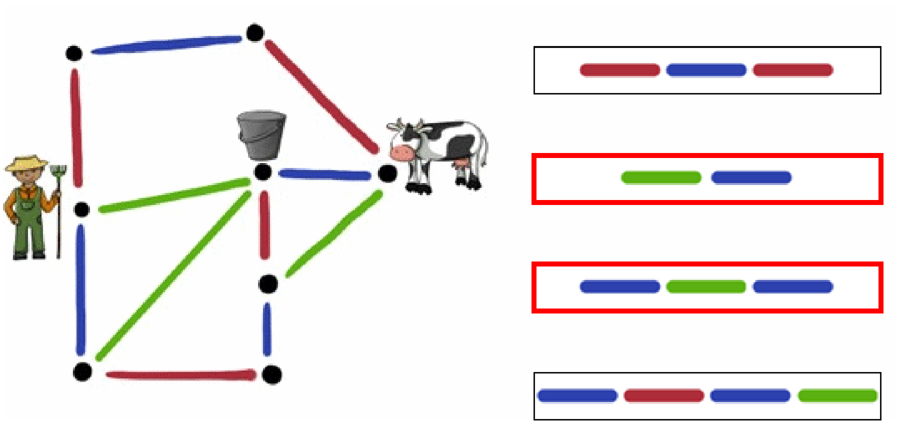

Figure 4 The task Coloured way

In the topic area **Sequences, relations, functions, tables, diagrams**, pupils have to discover quantitative and spatial relationships in reality and certain types of their systematic changes. They are able to work (through games and manipulation activities) with a particular set of objects according to an arbitrary criterion set before. The pupils are able to sort objects, things, elements in a given group according to one attribute (such as colour, shape, size, material, etc.) or find out a simple rule for creating a sequence of objects, things, elements and numbers.[11]

[11] http://www.statpedu.sk/files/documents/svp/1stzs/isced1/vzdelavacie_oblasti/matematika_isced1.pdf, p. 16

An example of such a task is (the Bobrík (easy) task called) *Little Creatures* which was solved correctly by most of children (12 of 13). In the task *Candies* (Fig. 5), that belongs to the most difficult tasks, only seven answers were correct.

Figure 5 The task Candies

In the topic area *Geometry and measurement*, pupils create spatial geometric objects according to particular rules, they become familiar with basic plane figures as well as with drawing them. They become familiar with basic properties of geometric figures, they learn to compare, estimate and measure the length. The Bobrík contest tasks as *Domino* (moderate difficulty), *Repaint* (difficult) and *Little Turrets* (difficult) belong to this area. Success percentage of these tasks was low (only 5 pupils out of 13 answered correctly). These tasks required good orientation in plane figures, knowledge of geometric shapes and imagination.

3 Conclusion

A teacher of 1st stage at St. Vincent Elementary school evaluates her pupils' participation in this contest: "In my opinion, the contest in Informatics education was especially important ... because children could test not only their knowledge of informatics but also of mathematics." Teachers of children who participated in this contest say that the problem is in deficiencies of pupils in reading comprehension. Hence, pupils often answered contest tasks incorrectly because they failed to understand the task’s text in the first place.

The informatics contest iBobor in its youngest category Bobrík brought many interesting tasks, which could inspire maths teachers to use them during their lessons. The contest showed which pupils' competences have to be developed more intensively.[12] Solving logical problems strengthens students' awareness of their abilities to use logical reasoning. It can capture even those students who are less successful. Pupils' skills acquired in the subject *Informatics Education* allow them to apply ICT also in other areas of primary education. Furthermore, development of pupils' digital literacy can support their other key competences.

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